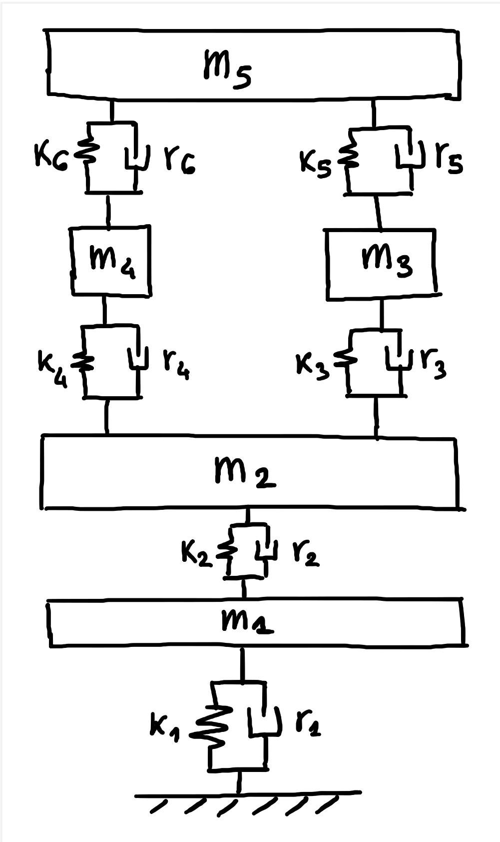
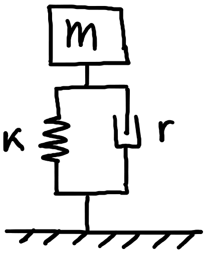
FRF AND EXCITATIONG TECHNIQUES

Introduction

Let's imagine that we have a quite complex mechanical system like the one depicted in the picture on the right. Usually when we are dealing with this kind of mechanical system, we always want to discover some feature of the system itself and generally we want to compute its natural frequency.



We know that the natural frequency of a simple system (1 degree of freedom), like the one in the picture below, made only of a mass, a damper and a spring can be computed as the square root of the ratio between the spring and the mass:



$$\omega_0 = \sqrt{\frac{k}{m}}$$

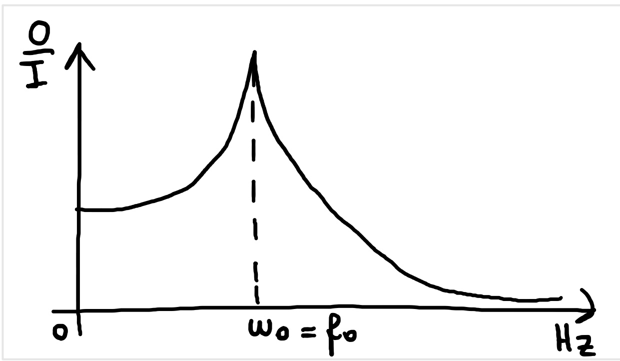
Since we are in the measurement course we want to understand if we can measure the natural frequency or at least take the measurements of the different parameters that are presented in the equation. So, the question that we now would like to answer is how we can compute the parameters required to compute the natural frequency (mass and spring). We know for sure that we can measure the mass thanks to a balance, for example. On the other hand, we are not so sure to be able to measure the stiffness of the spring since it will depend also on other aspects like how it is mounted. Maybe we can define it thanks to some experiments, but the value will never be precise for the reasons already explained. If then we would like to measure also the damper to know all the parameters of the system (even if it’s not necessary to compute the natural frequency), we can say that it is almost impossible measuring it.

Let’s make an example to clarify everything. Let’s consider a personal computer, and let’s image to hit the screen. We are now interested in measuring the parameters related to the screen that is vibrating. Of course, we can measure the mass of the screen by unscrewing it from the keyboard and using a balance. Maybe we can also compute the stiffness of the spring by studying for example the screw, but for sure we have no idea on how we can measure the damping.

So, we have understood that to determine the natural frequency, measuring the parameters of system is not effective. We can think of using a different approach and have a look at a plot called frequency response function.

Frequency response function

So, we can think of studying the natural frequencies by using another approach. We, in fact, can think of using a graph, a spectrum, which is the responses of the system to an input; this response is called frequency response function (FRF). The idea is that I excite the mechanical system at different input frequency, and I obtain as the output a response that changes with the frequency.



Once we have obtained the spectrum, we can identify the natural frequency of the system by identifying the frequency at which we have the peak.
The plot that we have obtained is a spectrum, as already mentioned. To be more précised it is the spectrum obtained by computing the ration between the spectrum of the output and the spectrum of the input.
Until now in the spectrum we have had in the vertical axis a specific quantity, an amplitude expressed in volt, for example, whereas now in the vertical axis we have a ratio and not a pure value. The ratio, as already mentioned, is obtained by dividing the output over the input. For this reason, once again, this plot can be considered as the combination of two spectra or at least the combination of two different behaviours.
Usually to obtain the output used to compute the FRF some experiments are carried out. These experiments consist in the excitement with a controlled input. These controlled inputs could be of different types; the main ones are:

1. Step sine (or harmonic excitation)
2. Swept sine
3. Impulsive
4. White noise

Step sine technique

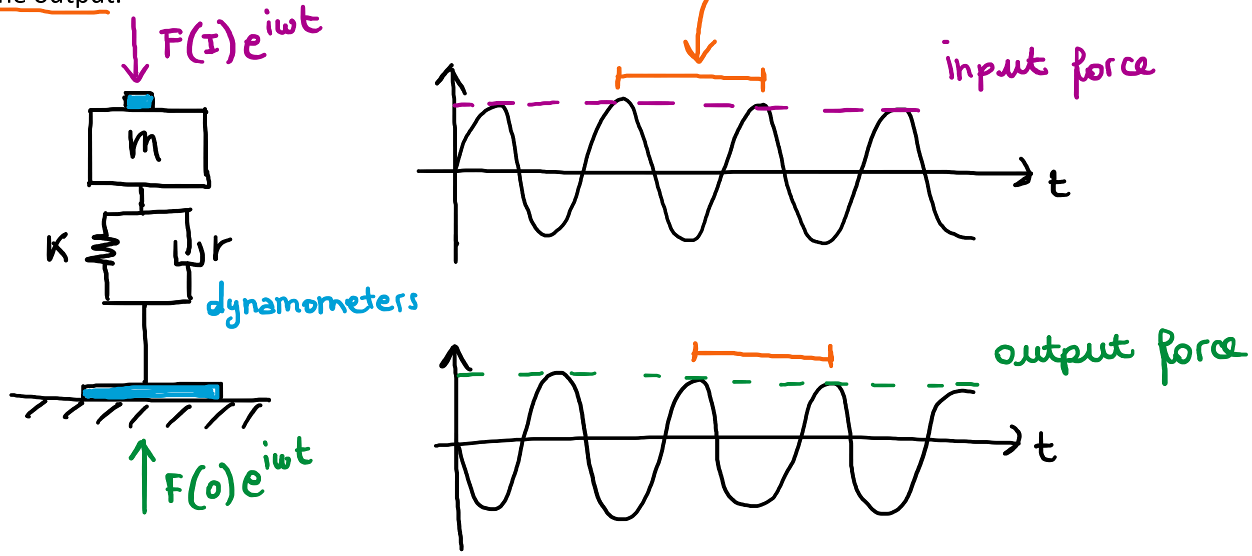
In this first case a simple sine wave is used to excite the system. Usually it is provided to the system thanks to the use of a vibrating machine on which the system is mounted. In order to study the response of different frequency, the frequency of the input must be changed following a discrete step previously defined. It's important to choose correctly the step of the frequency so that none frequency of interest is missed.
So, let's go back to our simple mechanical system with the spring and the damper; and let's understand how we can proceed in this case with the step sine approach. First, we must highlight that we could have two different case:

a) In this first case I know the input force
b) In this second case we don't know the input force. In this case a dynamometer placed on the mass is required so that we can measure and know also the input force

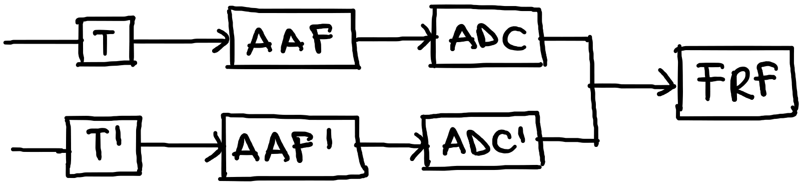
Anyway, in both cases thanks to a dynamometer that is usually placed on the ground we can acquire the output force. Now we have all the ingredients to compute the ratio between output and input.
As already stated, we need a force that changes its frequency and for this reason we usually have an exponential force and not a static one. So, we have a force that has a dynamic behaviour so that we can analyse and study all the interested frequency. So, to repeat once again we take measurement for different frequency; for each frequency we have an input and an output and then we can compute the ratio.
To sum up this approach we can say that we need to follow these steps:

- Choose a frequency variable input force
- Measure the output associated to that frequency
- For each frequency we must compute the ratio
- We finally can draw the plot

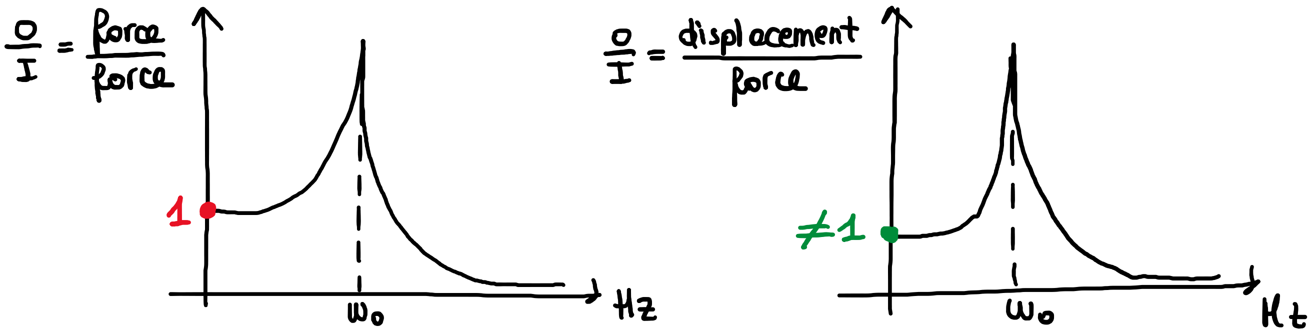
We can now draw once the scheme of our mechanical system with the plot of both the input and the output force. It's important to highlight that if the system is linear the period of the input signal is equal to the period of the output.



So now, with the introduction and the use of the FRF, we need two channels to carry out the measurement: one for the input and one for output; for this reason, we'll have two chains of measurement that are then connected together to have the FRF.



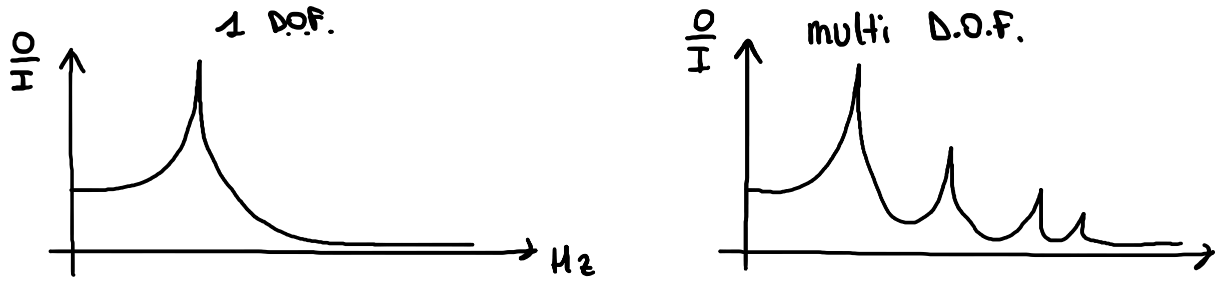
Until now we have discussed about the case in which the output is the force (the same "type" of the input), of course if we are interested in the displacement as the output and not in the force, we can think of placing a displacement transducer instead of the dynamometer. So, let's point out that the $F_o/F_i = 1$ for $\omega = 0$ as long as we use force over force, so in case of having displacement over force this value will be different from 1. We can now draw once again the scheme of our mechanical system with the plot of both the input and the output force. It's important to highlight that if the system is linear the period of the input signal is equal to the period of the output.



Under a theoretical point of view, I could use any amplitude of the force but by a practical and measurement point of view we must be sure that the input and the output are higher than the sensitivity so that they can be acquired correctly. Usually we could have two critical situations:

1. If we are near the $\omega = 0$ we could have forces too small to be acquired correctly (they could be affected by noise)
2. If we are near the peak, and so near the natural frequency ω_0, we could have force too high that could damage our system

In general, if our system has only one degree of freedom what we have done until now gives us a comprehensive and good result but when I have a multi degree of freedom we are not sure that following this path we'll have a reliable result. This is due to the fact that in this technique I need a frequency step that can actually affect our FRF: I could for example choose a frequency step that is bigger than the different between the first and the second natural frequency and so I could obtain a distorted value of the second natural frequency or I could complete lose the information of the second natural frequency. For this reason, we are going to introduce the other approaches that can help me calculate the FRF with a trade-off between accuracy, cost and speed (usually asked in the second midterm exam).

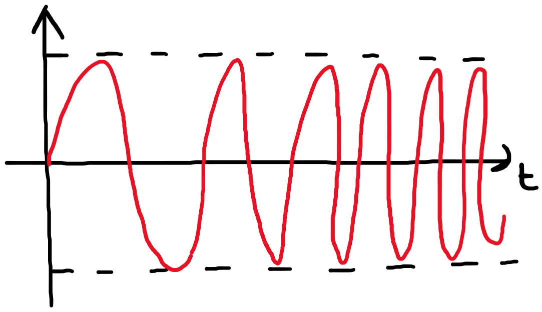


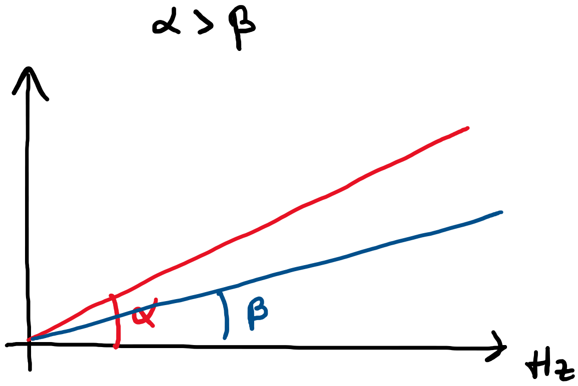
So, we have understood that his technique has a very big limitation, but on the other have it has a quite important advantage: in this approach we measure in steady state condition.

The technique that we have described until now and that it's usually used in a single degree of freedom system.

Swept sine technique

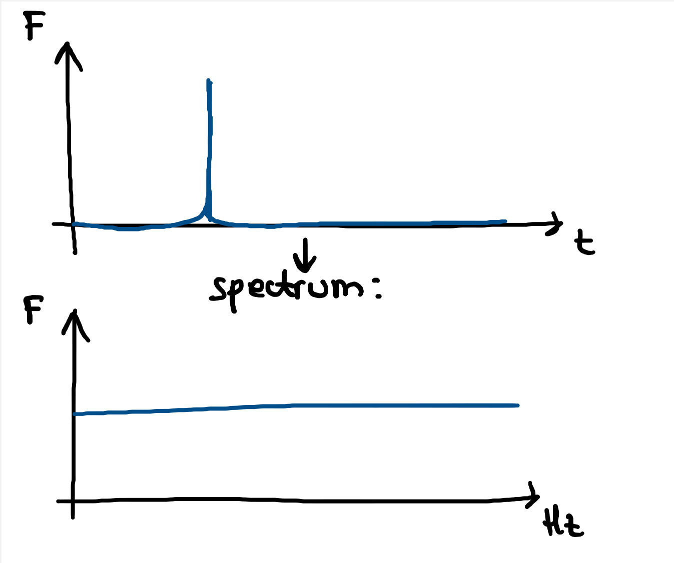
In order to overcome the limitation of the step sine approach we can think of use another method that is called swept sine. This technique is based on the idea of exciting the system with a harmonic function which frequency changes continuously in time as shown in the picture on the right in the time domain. In other words, we are exciting the system with a sine wave whose period keeps on decreasing. Thanks to this we remove the problem of the resolution frequency since we excite all the frequency of the range. The main drawback of this method is that we are never in the steady state condition and so we actually never reach the pure FRF since it is defined as steady state. In order to improve this aspect, we can think of setting a very low rate of increase.
The swept sine function is a function of the slope frequency and so in order to be very accurate we need to take a long frequency range which means having a decrease of the slope.
In order to have the best of both worlds usually a speed overview is done with a swept sine of intermediate level and then it's possible to go deeper in the details and go around the peaks under steady state with the step sine approach.





Impulsive technique

Until now we have talked about natural frequency, but we haven't defined it yet. A natural frequency is the frequency at which it is easy for the system to vibrate. We can use this definition and observation to develop a new technique to obtain the FRF. We can think of excite the system with an impulse since it leads us to cover all the frequencies in a very short time.
Its advantages are that it is the fastest, we don't have to make any choice about the resolution and so the resolution will be for sure $1/T$. The problem is that is not steady state at all, and it is only transient. It is a very effective technique, but it cannot lead us to have a steady state response function, it can be used to identify where the natural frequencies are.



The duration of the acquisition is an important parameter to highlight since it depends on the resolution itself. So, the key point is that I cannot acquire whenever I want but I have to respect a condition according to which the system has already finished to vibrate before I do another acquisition. So, if the frequency resolution is $0{,}01Hz$, I have to wait at least $100s$ before do another acquisition.

White noise technique

There is another technique that do not generate a harmonic function, but it generates some noise. A random signal, white noise, which covers a lot of frequencies. Usually this technique requires following averaging procedure.

Adding notes

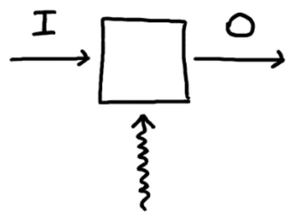
We could have two different problems:

a) Problems related to two channels = multiplex ADC that shares one converter for both channels can lead to small delay
b) Transducer = the dynamic behaviour of the transducer can be a problem when we take a ratio

	Advantages	Disadvantages
Step sine	• We are in the steady state condition	• Problem of the resolution (discrete frequency step) • Very time consuming
Swept sine	• no problem on frequency resolution since I excite all the frequencies	• we are never in the steady state condition • FRF almost never reached because we keep on increasing the frequency (this can be reduced by setting a very low rate of increase)
Impulse	• it's fastest • It is cheap	• It is transient and not steady state at all and so FRF is never reached • Maybe difficult to have an adequate energy input without damaging the system
White noise	• Easy to implement	• It is transient and not steady state at all and so FRF is never reached • Could not cover the frequency of interest because it’s a random signal

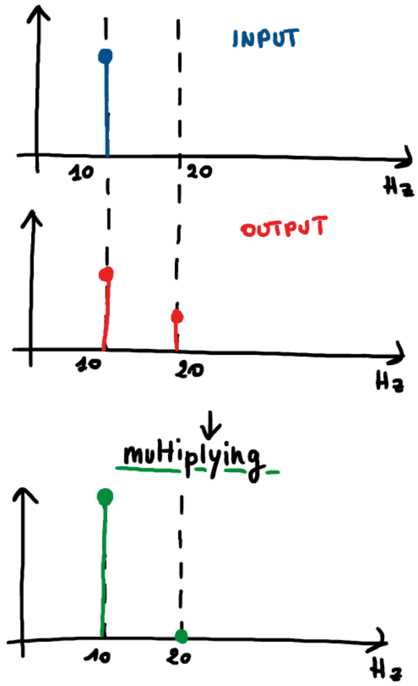
CROSS SPECTRUM AND COHERENCE

Introduction



In previous lessons we have studied and discussed about the FRF as the ratio between the output over the input. By doing this we have given as granted that we have only one input and, in particular, we have only the input in which we are interested. Actually, most of the time we have some spurious input coming from other phenomena, so, generally we have more than one single input. The bad thing is that most of the time we don't realise that we have these unwanted inputs.

Cross spectrum



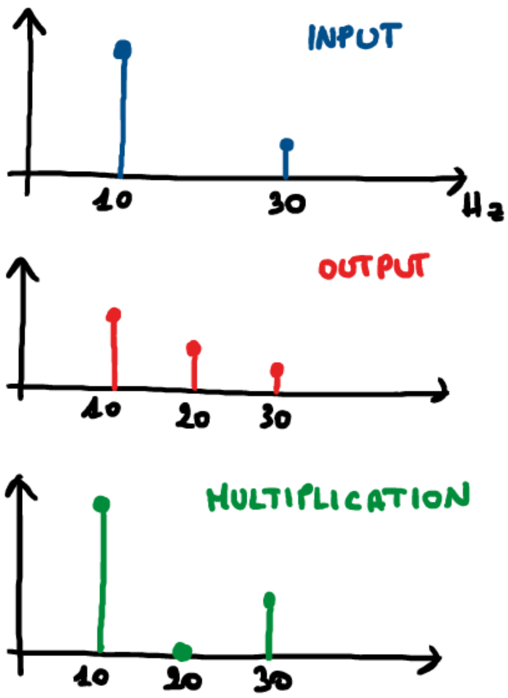
In order to understand how we can insert this aspect into the model that we have described until now let's image to have two spectra: one representing the input and one representing the output. In the input spectrum we have only one component at $10\ Hz$. In the second spectrum, the one that represents the output, we have 2 components: one at $10\ Hz$ and one at $20\ Hz$.
Since in the input we have only two components we can easily conclude that the component of the output at $20\ Hz$ is due to the unwanted and spurious input (disturbances or non-linearities). Since we have in the output this unwanted component that is not present in the input, we can say that the model that we have discussed until now is not perfectly correct. In addition to this we cannot think of doing the FRF because for the component at $20\ Hz$ we would have an amplitude divided by a 0 (we don't have any component at $20\ Hz$ in the input spectrum) and this would lead us to have an infinite value; this is, of course, not true and so we have for sure measured something wrong or at least we have forgotten to measure something at the frequency that is missing. So, now our question is how we can trace the presence of the component that is missing in the input but is present into the output? We can think of solving this problem by multiplying the two spectra. With this sentence we mean multiply frequency by frequency each component of the two spectra. By doing this we end up with a plot in which we have only the component related to the input. The output of this multiplication is a complex vector and it's called cross spectrum. The cross spectrum is a function allowing to get information on the "existing relation" between two analysed channels. By a mathematical point of view, it can be computed as the product between one spectrum (in our case the output spectrum) and the complex conjugate of another spectrum (in our case the complex conjugate of the input spectrum):

$$S_{AB}(f) = B^*(f) \cdot A(f)$$

where $B^*(f)$ is the complex conjugate of the input spectrum and $A(f)$ is the output spectrum; the fact that we have the complex conjugate comes to the fact that it helps on determining the phase delaying between one quantity and the other, in measurement it's not very useful but it's very important in the electrical fields. This formula, in fact, comes from the electricity field: if we think of this field, we would apply the cross spectrum on the voltage and the current and we would end up with the spectrum of the electrical power.

Coherence

Let's now image to have a second component in the input at $30\ Hz$ which amplitude is quite small; also, in the output spectrum we have this component at $30\ Hz$. In this case if we multiply the two amplitude of the components, we'll not have zero in the cross spectrum since none of the two components, in input and output are zero. In this case we cannot conclude anything, we cannot even compare this component with the one at 10 Hz and so we cannot say if it best or worse than that component. We need a further step to make this technique more effective in order to better understand when a component in the cross spectrum is reliable or not. To solve this problem, we need to normalise the cross spectrum so that we can have a sort of alignment. By doing this we are also able to understand how good or how bad is the output that we obtain with respect to the input. To normalise we need to:



$$0 \leq \gamma_{AB}^2(f) = \frac{|S_{AB}(f)|^2}{S_{AA}(f) \cdot S_{BB}(f)} \leq 1$$

Where S_{AB} is the cross spectrum, obtained by using the formula previously provided, S_{AA} is the autospectrum of the input and S_{BB} is the autospectrum of the output. Let's remember that the autospectrum is a real value obtained as the product between the spectrum and the complex conjugate of the same spectrum:

$$S_{AA}(f) = A(f) \cdot A^*(f)$$

$$S_{BB}(f) = B(f) \cdot B^*(f)$$

We can define γ_{AB}^2 as the coherence. This value is used to understand when we have coherence between the input and the output. In other words, it's a dimensionless value that is used to determine how two signals are linearly related. We can point out that the denominator of the formula represents the maximum energy that we can have. In addition to this we can also say that the autospectrum and the coherence are real values whereas the cross spectrum is a complex value; anyway, all these quantities are function of the frequency. It can be a value between 0 and 1: when we have 1 we have maximum energy and so we have the perfect correspondence between the input and the output, on the other hand when we have 0 we don't have any energy and this means having no correspondence between the input and the output. The coherence is always equal to 1 when computed on a single time record; let's try to understand why with a demonstration:

$$\gamma_{AB}^2(f) = \frac{|S_{AB}(f)|^2}{S_{AA}(f) \cdot S_{BB}(f)} =$$

$$= \frac{|S_{AB}(f)|^2}{[A(f) \cdot A^*(f)] \cdot [B(f) \cdot B^*(f)]} =$$

$$= \frac{|S_{AB}(f)|^2}{S_{AB}(f) \cdot S_{BA}(f)} =$$

$$= \frac{S_{AB}^*(f) \cdot S_{AB}(f)}{S_{AB}(f) \cdot S_{BA}(f)} =$$

$$= \frac{S_{AB}^*(f)}{S_{BA}(f)} =$$

$$= \frac{S_{BA}(f)}{S_{BA}(f)} = 1$$

For this reason, it must be always computed using the averaged cross spectra and auto spectra. Let's now try to understand why the cross spectrum helps us to understand if there is a linear relationship between input and output:

$$S_{AB} = A^*(f) \cdot B(f) =$$

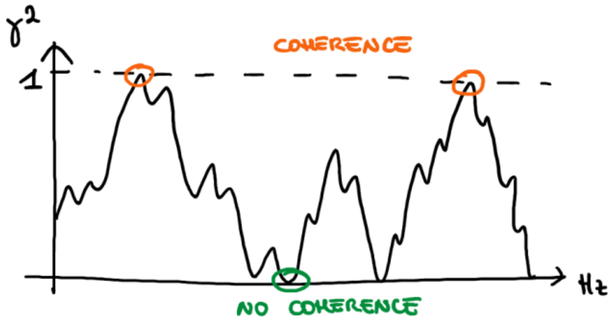
$$= \left[|A(f)|e^{i\varphi_A}\right]^* \cdot \left[|B(f)|e^{i\varphi_B}\right] =$$

$$= |A(f)|e^{-i\varphi_A} \cdot |B(f)|e^{i\varphi_B} =$$

$$= |B(f)||A(f)|\, e^{i\varphi_B - i\varphi_A} =$$

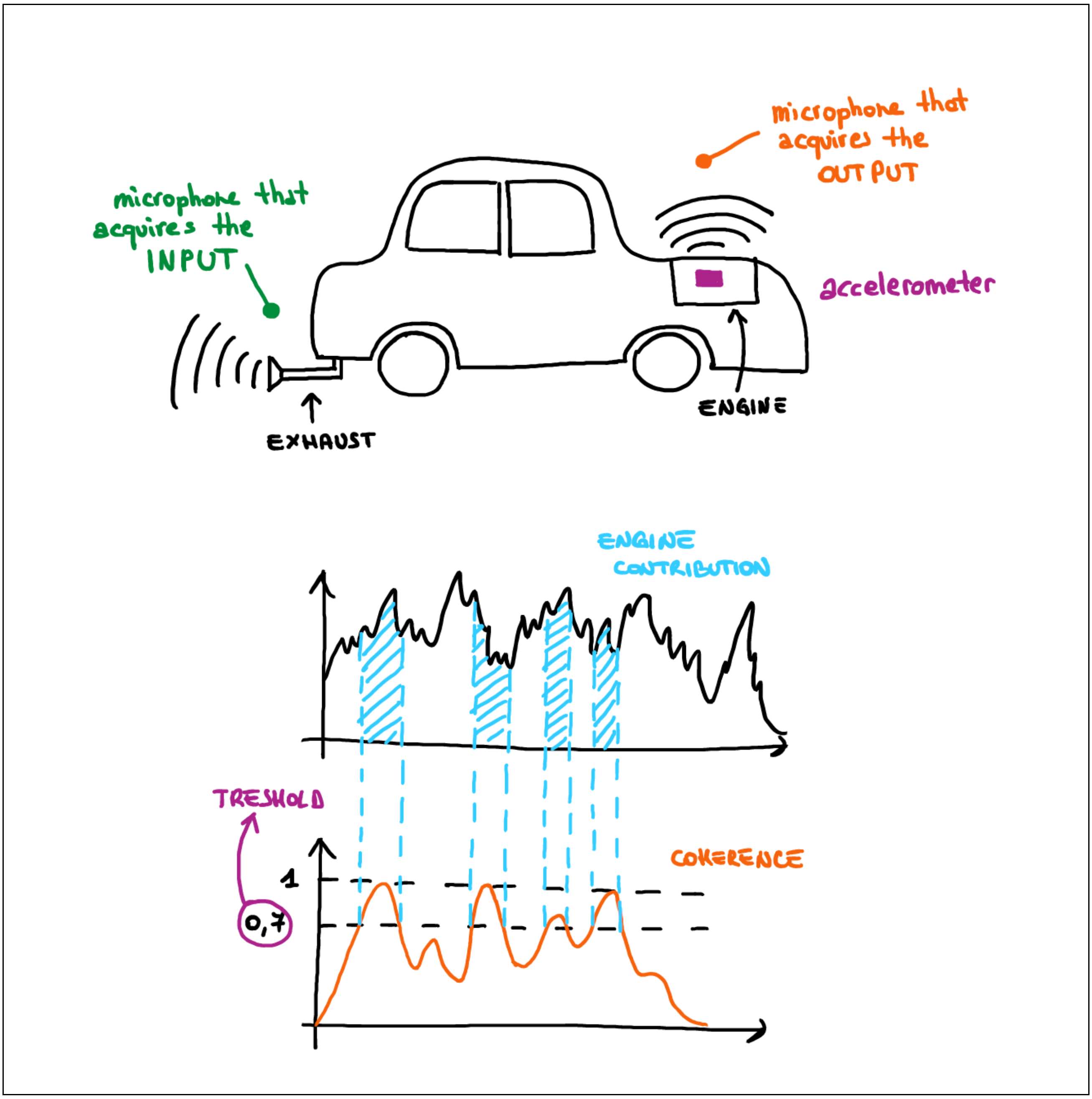
$$= |B(f)||A(f)|\, e^{i\varphi_{AB}}$$

We can now depict the coherence in a plot to better understand in which frequency we have coherence and in which we don't.



<u>Example</u> (something similar can be asked in the exam): let's imagine having a car with an engine and the exhaust[1]. Someone complains that the car is too noisy, so we need to work on it to reduce the question; the problem is that we don't have unlimited money but a fix quantity that allows us to act only on one of the two components. So, we can choose of working on the engine or on the exhaust, but not on both of them. In order to understand on which part, I'm supposed to work we must first discriminate and identify what of the two components is the most responsible one for the noise. To do that we must be able to understand what part of the total noise that I can acquire comes from the engine and what part come from the exhaust. So, we of course, the coherence will help me to make this discrimination. The problem is trying to understand the coherence of what? What are the signals on which I can then compute the coherence? What are the inputs?
Talking about the output we can put a microphone near the car and take the measurement. A more difficult choice would be the place in which acquire the input; at first, we can think of putting a microphone near the exhaust and consider it as the input or, vice versa, put a microphone near the engine. The problem of both these acquisitions is the fact that both of them will be influenced by the other one. So, in this case we will have always a pollution of what comes from the other component. To solve this problem, we can think of not using a microphone but using an accelerometer to measure the vibration. We can place the accelerometer in the engine so that the vibration will less influenced by the exhaust. We can then take the coherence between the accelerometer and the microphone. We can fix a threshold and say that if the coherence is bigger than this threshold the vibration (and so the noise) would come from the the engine, otherwise form the exhaust.

[1] Tubo di scarico/scappamento

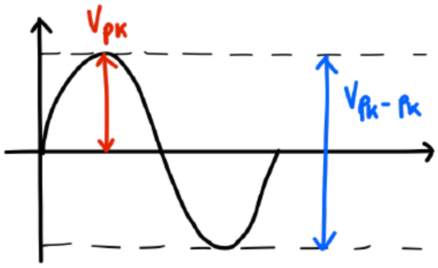


microphone that acquires the OUTPUT
microphone that acquires the INPUT
accelerometer
EXHAUST
ENGINE
ENGINE CONTRIBUTION
TRESHOLD
1
0,7
COHERENCE

ROOT MEAN SQUARE

Introduction

Let's recall some basic nomenclature that will be extremely useful in this chapter:



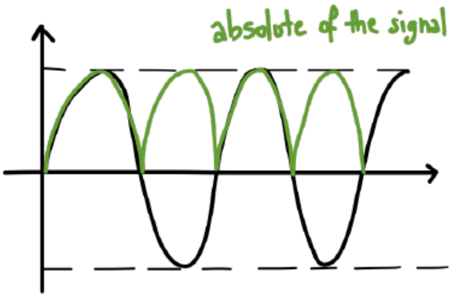
- Volt peak V_p= amplitude of the signal
- Volt peak to peak V_{pk-pk} = the difference between the minimum peak and the maximum peak.
- Mean value = it is the mean value of the signal, but, as we have previously stated it can be a tricky value, in fact if we think to the current we know that it has a trend of a sine wave as the one depicted in the picture. In this example the mean value (by a mathematical point of view) is of course 0 but if we put the finger into the socket, we won't' feel anything (even though the mean value is zero) since we'll feel the current. So, we conclude that this value is not a very reliable one to describe a dynamic signal.

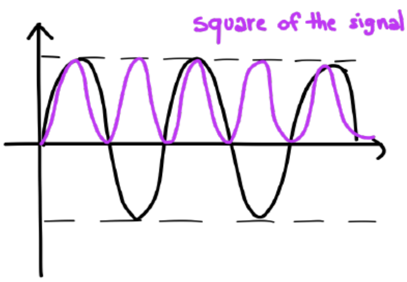
Though, if we are dealing with a signal that has a dynamic behaviour, we must use another concept that is related to the mean but that it's not the mean value. This concept is the RMS which stands for the Root Mean Square. We can consider the RMS for the dynamic behaviour as the equivalent of the mean value of a static signal. Let's try to understand how we reach the conclusion to use the RMS.

Approaches to handle a dynamic signal

In addition to this we must highlight that when we take into account a dynamic signal we are interested in both the negative and the positive part; so for us the positive and negative part have exactly the same weight and so we must find a way to quantify the phenomenon taking into account also this aspect. We can think of different ways to handle this aspect:

1. Absolute value = the easiest way that we can think of is taking the absolute value and then take the average this signal. In this case of course we end up with a value different from 0.
2. Square of the original wave = another approach that we can follow is taking the square of the original wave, this implicitly brings everything in the positive side.





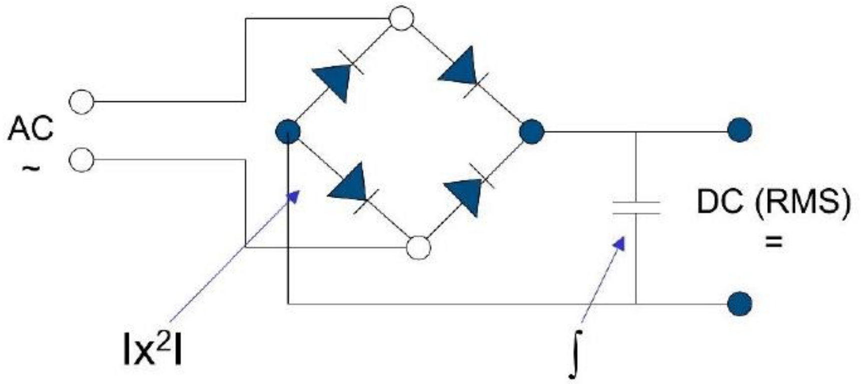
3. Electrical prospective = we can also consider an electrical prospective. In electrical fields, for alternate quantities, for instance AC voltage, the parameter for the measurement of the amplitude is the RMS. In fact, if we have a time varying signal at the inlet of a bridge diodes we'll have as the output a flat signal thanks to the rectification, since it "straightens" the direction of current. The bridge diodes is called rectifier and it's an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. The flat output represents the RMS voltage. We can use this definition that comes from the electrical fields also in out acoustic and mechanical fields since the transmission of the power along the hearing chain is very similar to the electrical transmission. Thus, as in electricity, also in acoustic the synthetic values of the AC quantities are expressed as RMS.

   The complete circuit for the RMS comprehends also a condenser (or capacitor) that cumulate the electrical charges. Since it is a analogical system it is not able to store in a memory the values computed each time and when the circuit is unplugged the information is definitely lost. Thus, the condenser required a certain amount of time to reach the steady state condition. This amount of time is called time constant τ and it is defined as the time necessary to reach the 63% of the station value (which is the RMS value) after a step-excitation. The trend following during the charge will be always exponential. If we increase the capacity of the condenser, it will be able to store more data but its promptness decreases, so we need to define some standard so that we can compare the results.

Now day we use digital device that have no problems in term of promptness but we still need to compare our results with the analogic standards present in literature, so to recreate exactly the behaviours that we would have in the analogical device with a digital one we change the length of the window. In the standards three types of window have been defined:

- Fast = 0,125 s: it was chosen since it's the limit thanks to which we identify the difference between ramble and eco sound.
- Low = 1,00 s
- Impulse

All of these three time-constants are from a certain point a low pass filter. In general, if the time constant is low the RMS goes up and goes down faster, whereas if it's high the RMS goes up and down low. An important aspect to cover is do I need such a huge amount of data? No because, as long as my window is a low pass filter, since it eliminates all the fluctuation, all the last part of the bandwidth is useless so instead of keeping all the signal we can think of taking into account only the first part. So, we can reduce the number of data without losing anything.



RMS

Let's now focus our attention onto the last approach that we have just defined according to which we consider the RMS as the indicator of the amplitude for dynamic quantities. The English definition of the RMS is focused on the mathematics of the problem: root, mean and square. On the other hand, the Italian or Latin definition of this parameter are a little bit more meaningful since it is called *valore efficace* that in English would sound approximately like *effective* value that is a value proportional to the effect induced by the AC phenomenon. The formula of the RMS can be defined in two different ways, following two difference points of view:

- Mathematical point of view:

$$RMS = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$

- Measurement point of view:

$$RMS = \sqrt{\frac{\sum x_i^2}{N}}$$

Since we are computing $x^2(t)$ we can say that:

- The signal becomes always positive
- The mean value of the signal is offset in the positive semi-plane

Let's now notice some key points of the RMS:

1. The formula just provided is valid only for the sinewave
2. The RMS is an average quantity, not an instantaneous value, and for this reason it doesn't make any sense to calculate the RMS of one sample or even on a period much shorter than the time of the cycle of an AC wave.
3. The RMS of a DC value is the DC value itself
4. The RMS is always non-negative, so it will always be positive or at least equal to zero
5. The RMS of a sine is equal to the peak value divided by $1/\sqrt{2}$

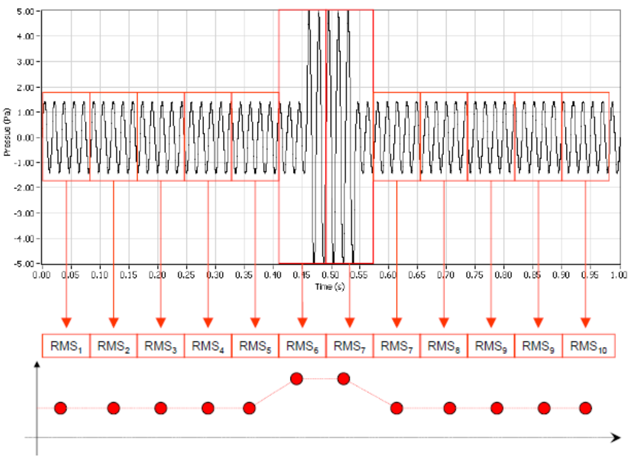
It's important to highlight that the RMS of a time changing signal can be not univocal due to 2 main reasons:

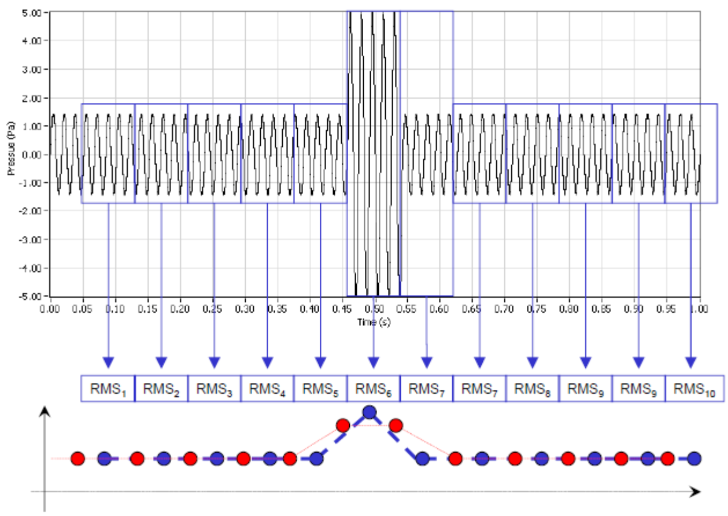
a) Different processing starting point
b) Different length of the window

In order to understand better these two aspects, let's take into account the example of what happen in a burst, which is a typical varying signal that have an almost instantaneous increase of amplitude that is maintained for a certain amount of time, and then comes back to lower

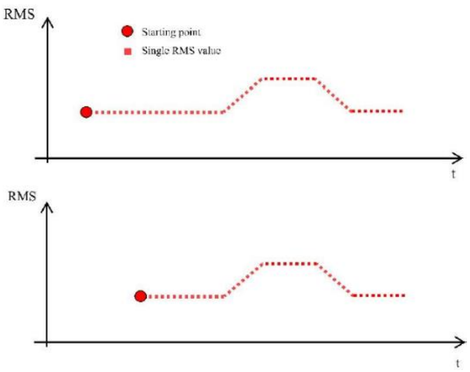
Different processing starting point values

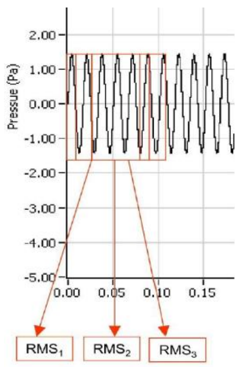
Let's assume that the calculation of the RMS is made by processing one discrete window per time. Depending on what region of the signal fells into the window we can compute different values of RMS, of course the region of the signal fells into the window depends on the starting point. In the following two imagine I have a clear example of an RMS of a burst computed with one discrete window per time with different starting points.





The solution to solve this problem is choosing a moving window which is a window that is moving, and which start at each sampling points of the signal. Thanks to this technique after a certain amount of time depending of which sample, we start, all the data will be the same and so reliable one to the other. Of course, this process can be done just in the digital since we are dealing with a discrete domain and not in analogical since we have a continuous domain. The main drawback of this process is the amount of information and the computing process.





Different length of the window

Regarding this second problem we must say that is has not been technically solved, but simply it was agreed that some standardized interval had to be chosen. To be more precise a time constant has to be chosen. The standard has established 3 time constants, the one already introduced.

<u>Exercise</u> (compitino): we have a burst (a sine with a short duration) with a duration of $1s$ as the one shown in the picture. Its amplitude is given and it's $V_{pk} = 20m/s^2$. Knowing that the amplitude of the noise is $V_{noise} = 1\,m/s^2$, $s$ketch qualitatively the RMS time history for three different time constants: $125ms$, $250ms$and $100ms$.

In order to proceed with the sketch, we need first to compute the RMS value of both the noise and the burst. To do that we must multiply each peak per $1/\sqrt{2}$:

$$RMS_{noise} = \frac{V_{pk}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0{,}707 \qquad RMS_{burst} = \frac{V_{pk}}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14{,}14$$

Now, regarding the burst we must compute the value reached when the constants is reached:

$$63\% \cdot RMS_{burst} = \frac{63}{100} \cdot 14{,}14 = 8{,}91$$

Now we must draw the RMS time history in the three different cases:

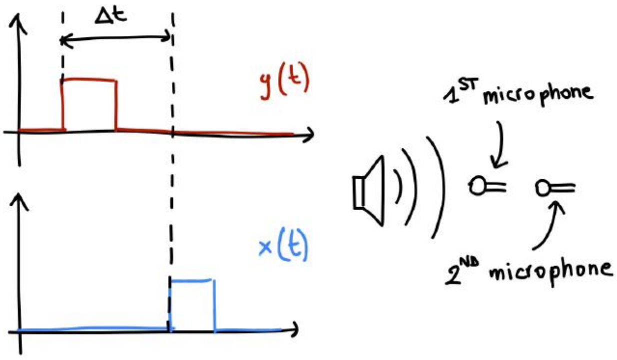
- Since we know that the first time constant is $0{,}125s$ and we know that the duration of the burst is 1s we can conclude that we'll need 1 period to reach the steady state condition
- if we have a time constant of $250ms$ we'll need 2 period to reach the steady state condition
- if we have a time constant of 1000ms we'll never reach the steady state condition

So, in conclusion, we can say that the number of periods required to reach the steady state condition can be computed as the ratio between the total length of the burst and the time constant. The final plot will be:

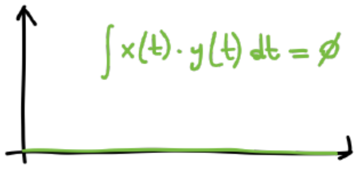
CROSS-CORRELATION AND AUTOCORRELATION

Cross-correlation

Let's imagine having two signals that have a trend like the one depicted in the following picture. We can imagine that they are signals coming from two different transducers: we have, for example, a sound speaker that emits a sound then we have two microphones. At first, of course, the sound reaches the first microphone and then it reaches the second one. We would like to know how long the sound takes to travel from the first microphone to the second one; in other words, we want to compute the time difference Δt between the two signals. To compute and know this delay of signal we can think of using a trigger but usually this is not done due to the fact that we could have a synthetic or a real sound and so this approach is not so effect; we want to find a more robust algorithm.

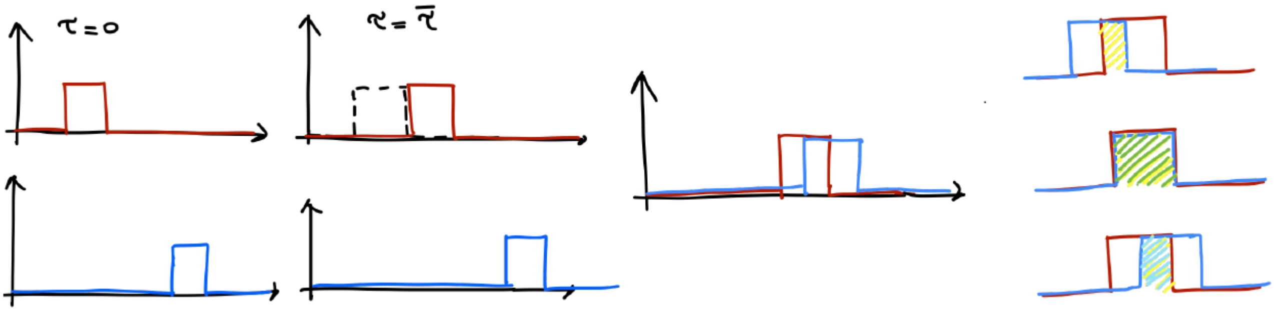


To compute the time delay between the two signals we can think of multiplying the signals and then doing the integral:



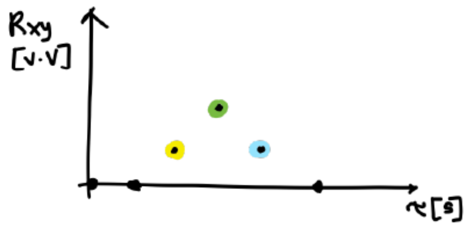
$$\int x(t) \cdot y(t) dt$$

The problem is that in the example just depicted the result of this operations will be zero since the multiplication between the two signals is zero. Thus, we need another strategy. We can think of shifting the first signal on the right side of a quantity τ which is a time dimensional value. Thanks to this strategy we end up with a value different from zero. Let's try to explain why through the example itself. Let's choose first a certain $\bar{\tau}$. Then for a $\tau = 0$ we have that $y(t)$ is in the same position as before and so the integral of the product between the two signals is zero. Then for $\tau = \bar{\tau}$ we have the signal $y(t)$ is moved forward of $\bar{\tau}$, since they two signals are not superimposed the result is still zero. Then for $\tau = 2\bar{\tau}$ we are still in the same situation and so the result is still zero. At a certain point the two signals will be superimposed and so the results will be a value different from zero.



To be more precise, by a mathematical point of view we are computing the following value that we call correlation:

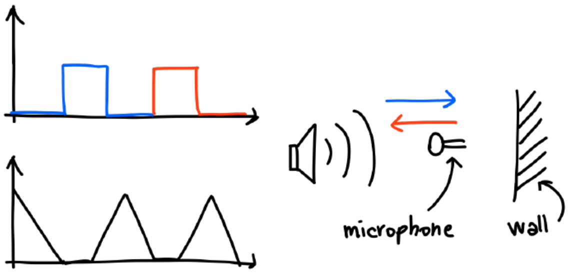
$$R_{xy} = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \cdot y(t + \tau) dt$$



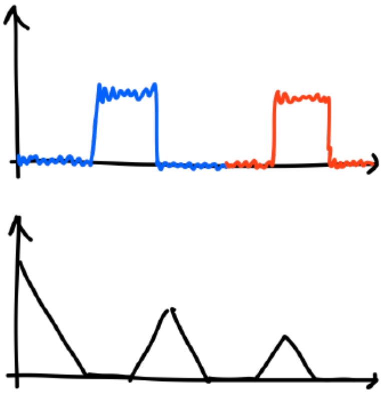
This concept gives us an idea on how the correlation between two signals is. By identifying the position in which we have the maximum value we can obtain the Δt. In the plot we can see that we don't have any correlation between the two signals until a certain point.

So, we have understood that the cross correlation can be used to determine the time delay between two signals.

Autocorrelation

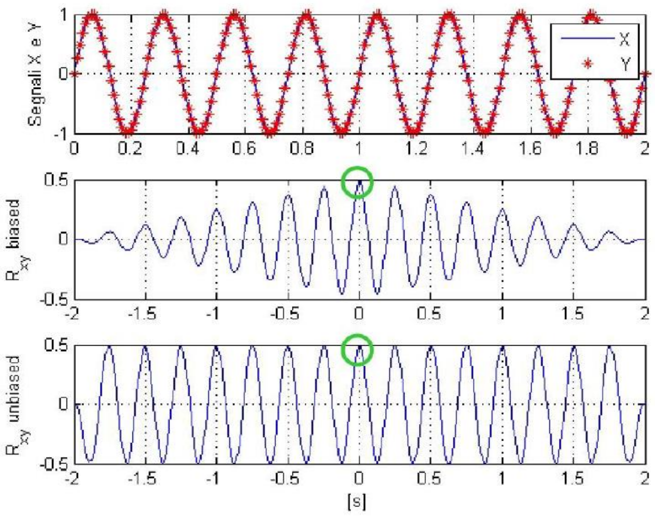


If now we imagine to eliminate the second microphone and add a wall after the first microphone we have a different situation: in this case we don't have two different signals but we have the same signals that hit the wall and comes back to be acquired a second to the microphone. In this case we could have a plot like the one shown in the picture. If we apply the concept described until now in this case, we end up with multiplying the signal by itself; this operation is called autocorrelation (it is a sort of artefact because I copy the original signal one and shift the first on). By a mathematical point of view the autocorrelation is defined as:

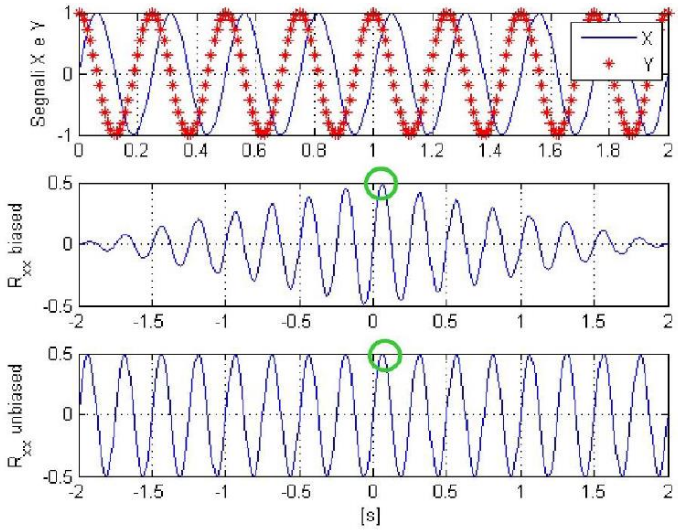


$$R_{xx} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) \cdot x(t+\tau)dt$$

As opposed to the correlation in which we started with complete not superposition, in the autocorrelation we start with complete superimposition and then we decrease.



Graphical example of Auto-correlation

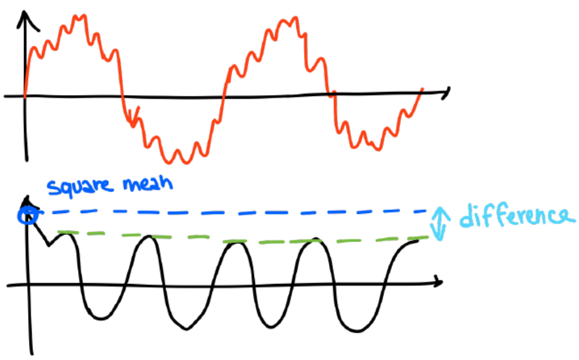


Graphical example of Cross-correlation

Autocorrelation: practical example

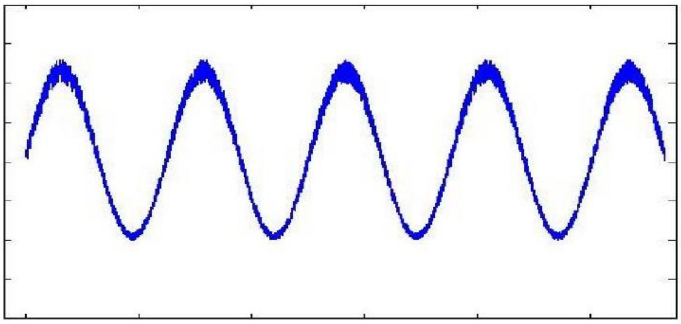
The autocorrelation can be used for several reason and aims:

- Determine the periodicity of a particularly noise signal
- Determine how noisy is the signal
- Extract a harmonic signal free from noise if we have a stable signal affected by the noise. In order to do this, we must follow two steps:
    1. Apply an unbiased autocorrelation: after the unbiased autocorrelation we end with two main problems:

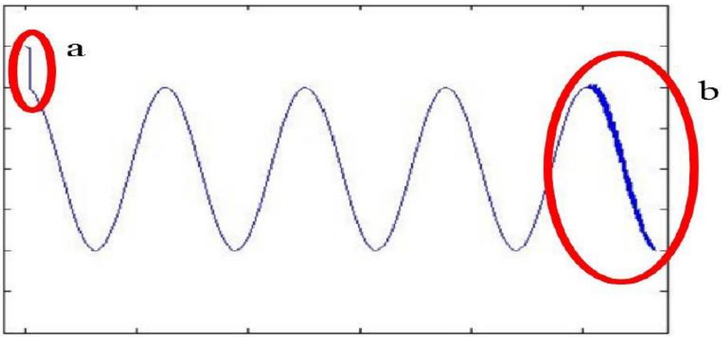


a. Problem A is generated by the complete overlapping of the two functions. In this case the noise of the two functions are completely overlapped and so the value of the integral is increases.
b. Problem B is due to the fact that the noise begins increasing because the length of the integration is not enough to reduce significantly its effect

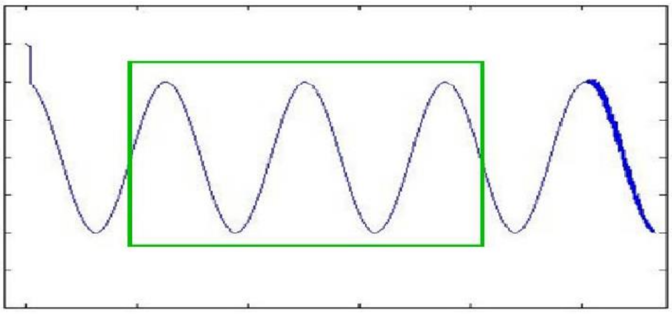
2. Extract a region not affected by the noise



Example of signal affected by noise



Unbiased auto-correlation on a signal affected by noise
and
Visualization of the two issues after unbiased auto-correlation



Extraction of the central part of the unbiased auto-correlation

Biased and unbiased correlation

In the previous definition provided of the correlation the factor of normalisation $1/T$ is indipendet from the variable τ: in this case we talk about the biased correlation. In this case if we are dealing with a finite signal, we have that as soon as τ increases the value of the biased correlation R_{xx} decreases. On the other hand, we can use the unbiased definition in which we take into account also τ in the normalisation factor $(\frac{1}{T-\tau})$.

Transmission of data

Let's now try to focus our attention to one topic that we have introduced at the beginning of the course: what is a data bus? How the data are transmitted? The transmission changes with the respect to the type of measurements:

- Analogue: the data are transmitted as voltage proportional to the value, this transmission is able thanks to the wire
- Digital: in this type of transmission I use wires to transmit voltage but, in this case, we don't have a proportional voltage since the voltage has only two levels.

We must not believe that digital transmission is perfect. In fact, after we have 1 the voltage goes back to 0 but this is because we are sending a 0 or it is only to be considered as a separation to next transmitted 1? Thus, we need two types of information and so we need two couple of wires more that sends a service information: the start and the end of the acquisition. The second service information that is required is the Δt that divides every single information. This is still a little bit ambiguous because we now know how to indicate both the start and the stop but how can we know that the stop is a stop and not a start? We can solve this problem by adding a second *up* to indicates the stop so that we do not have any misunderstanding. We have now understood that the language is an important aspect and it's called protocol. With the same bus we can transmit different protocols.

So, we have defined the bus as a physical way in which we transmit a signal, the protocol is the meaning attributed to the bus. We need of course some agreement to have and use the same language.

We also need to try to avoid having too much wires and for this reason we can think of having a unique combination of data that works as out trigger so that we can eliminate two wires associated with the start of the signals. We can also think of doing the same thing for the end of the signal so that we end up with only having two wires. This transmission is called serial transmission. It was the first protocol used, the same used nowadays in the USB. By a general point of view this series are called hand shake between the transmitter and the receiver.

Difference between echo and ramble

We define echo as a reflection of sound that arrives at the listener with a delay after the direct sound. The delay is directly proportional to the distance of the reflecting surface from the source and the listener. Echo with respect to the ramble is able to distinguish the two sounds because they arrive to the humans' ears after $125\ ms$, whereas ramble doesn't distinguish the two sounds since the second sound arrives before $125ms$ and so the ears are not able to make this distinguish.

COMPITINO questions:

- I have a signal with an autocorrelation explain if the signs, is noisy or not noisy and why
- Given this function compute the correlation