SISTEMI A PIU' GRADI DI LIBERTA'

Energia cinetica in coordinate fisiche

$$E_c = \frac{1}{2}\underline{\dot{y}_m}^T[M_f]\underline{\dot{y}_m}$$

$$\underline{\dot{y}_m} = \begin{Bmatrix} \underline{\dot{y}_{m1}} \\ \underline{\dot{y}_{m2}} \\ \vdots \\ \underline{\dot{y}_{mnc}} \end{Bmatrix} \qquad \underline{\dot{y}_{m1}} = \begin{Bmatrix} v_1 \\ \omega_1 \end{Bmatrix}$$

$$[M_f] = \begin{bmatrix} [M_{f1}] & 0 & 0 & 0 \\ 0 & [M_{f2}] & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & [M_{fnc}] \end{bmatrix}$$

$$[M_{f1}] = \begin{bmatrix} m_1 & 0 \\ 0 & J_1 \end{bmatrix}$$

Energia cinetica in coordinate indipendenti

$$\underline{\dot{y}_m} = \underline{\dot{y}_m}(q) = \left[\frac{\partial \underline{y_m}}{\partial \underline{q}}\right]\underline{\dot{q}}$$

$$E_c = \frac{1}{2}\underline{\dot{y}_m}^T[M_f]\underline{\dot{y}_m} = \frac{1}{2}\underline{\dot{q}}^T\left[\frac{\partial \underline{y_m}}{\partial \underline{q}}\right]^T[M_f]\left[\frac{\partial \underline{y_m}}{\partial \underline{q}}\right]\underline{\dot{q}} = \frac{1}{2}\underline{\dot{q}}^T[M]\underline{\dot{q}}$$

$$[M] = \left[\frac{\partial \underline{y_m}}{\partial \underline{q}}\right]^T[M_f]\left[\frac{\partial \underline{y_m}}{\partial \underline{q}}\right] = [M(q)]$$

Energia potenziale in coordinate fisiche

$$\mathrm{V} = \frac{1}{2}\underline{\Delta l_k}^T[K_{\Delta l}]\underline{\Delta l_k} + \underline{p}^T\underline{h_p}$$

$$\underline{\Delta l_k} = \begin{Bmatrix} \Delta l_1 \\ \Delta l_2 \\ \vdots \\ \Delta l_{n_k} \end{Bmatrix} \qquad \underline{h_p} = \begin{Bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{n_p} \end{Bmatrix}$$

$$[K_{\Delta l}] = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & k_{n_k} \end{bmatrix} \qquad \underline{p} = \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_{n_p} \end{Bmatrix} = \begin{Bmatrix} m_1 g \\ m_2 g \\ \vdots \\ m_{n_p} g \end{Bmatrix}$$

Energia potenziale in coordinate indipendenti

$$\underline{\Delta l_k} = \underline{\Delta l_k}(q)$$

$$\underline{h_p} = \underline{h_p}(q)$$

$$V = \frac{1}{2}\underline{\Delta l_k}^T(q)[K_{\Delta l}]\underline{\Delta l_k}(q) + \underline{p}^T\underline{h_p}(q)$$

Energia dissipativa in coordinate fisiche	Energia dissipativa in coordinate indipendenti
$D = \frac{1}{2}\underline{\dot{\Delta l}_r}^T [R_{\Delta l}]\underline{\dot{\Delta l}_r}$ $\underline{\dot{\Delta l}_r} = \underline{\dot{\Delta l}_k} = \begin{Bmatrix} \dot{\Delta l}_1 \\ \dot{\Delta l}_2 \\ \vdots \\ \dot{\Delta l}_{n_k} \end{Bmatrix}$ $[R_{\Delta l}] = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & r_{n_s} \end{bmatrix}$	$\underline{\dot{\Delta l}_r} = \underline{\dot{\Delta l}_r}(q) = \left[\frac{\partial \underline{\Delta l_r}}{\partial \underline{q}}\right]\underline{\dot{q}}$ $D = \frac{1}{2}\underline{\dot{\Delta l}_r}^T [R_{\Delta l}]\underline{\dot{\Delta l}_r} = \frac{1}{2}\underline{\dot{q}}^T \left[\frac{\partial \underline{\Delta l_r}}{\partial \underline{q}}\right]^T [R_{\Delta l}] \left[\frac{\partial \underline{\Delta l_r}}{\partial \underline{q}}\right]\underline{\dot{q}} = \frac{1}{2}\underline{\dot{q}}^T [R]\underline{\dot{q}}$ $[R] = \left[\frac{\partial \underline{\Delta l_r}}{\partial \underline{q}}\right]^T [R_{\Delta l}] \left[\frac{\partial \underline{\Delta l_r}}{\partial \underline{q}}\right] = [R(q)]$
Componente lagrangiana delle forze in coordinate fisiche	**Componente lagrangiana delle forze in coordinate indipendenti**
$\delta^* L = \underline{F}^T \delta^* \underline{y_f}$ $\underline{F} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{n_f} \end{Bmatrix}$ $\underline{y_f} = \begin{Bmatrix} y_{f1} \\ y_{f2} \\ \vdots \\ y_{n_f} \end{Bmatrix}$	$\underline{y_f} = \underline{y_f}(q)$ $\delta^* \underline{y_f} = \left[\frac{\partial \underline{y_f}}{\partial \underline{q}}\right]\delta^* \underline{q}$ $\delta^* L = \underline{F}^T \delta^* \underline{y_f} = \underline{F}^T \left[\frac{\partial \underline{y_f}}{\partial \underline{q}}\right]\delta^* \underline{q} = \underline{Q}^T \delta^* \underline{q}$ $\underline{Q} = \left[\frac{\partial \underline{y_f}}{\partial \underline{q}}\right]^T \underline{F} = \underline{Q}(q)$