POSSIBLE QUESTION ABOUT THE SECOND HALF OF THE EXAM

1. Cross correlation and autocorrelation, definition and practical examples

Imagine having two signals in time domain, $x(t)$ and $y(t)$. These signals can be obtained, for example, by considering a sound speaker and two different microphones placed at different length form the sound speaker itself. The cross correlation is defined as:

$$R_{xy}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) \cdot y(t+\tau) dt$$

Whereas in order to define the autocorrelation let's imagine having one signal that is acquired in two different moment. We have, for example, a sound speaker, a microphone and a wall. The sound speaker emits some sounds that intersect the microphone; thus, the device acquires this sound, then the sound wave hits the wall and goes back to the same direction from which it came from, it is then acquired once again by the microphone. At the end we have acquired the same signal in two different times. In this situation we can compute the autocorrelation of the signal $x(t)$ as:

$$R_{xx}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) \cdot x(t+\tau) dt$$

The cross correlation can be used to determine the time delay between the two signals related to the same phenomena whereas the autocorrelation is often used to determine the periodicity of a particularly noise signal or to understand is how noisy is a signal.

2. Biased and unbiased autocorrelation

When we talk about the biased autocorrelation, we are thinking of a normalization factor that is independent from τ, since it's equal to $1/T$:

$$R_{xx}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) \cdot x(t+\tau) dt$$

In this situation, in the biased autocorrelation, if we have a finite length signal having a normalization factor that depends only on T means that the value of autocorrelation $R_{xx}(t)$ decreases with the increase of τ. So, the biased autocorrelation tends to 0 for big time delays because it is divided by big numbers. On the other hand, when we talk about the unbiased autocorrelation, we have that the normalization factor depends also on τ and not only on T, thus $\hat{R}_{xx}(t)$ is kept constant:

$$\hat{R}_{xx}(t) = \lim_{T\to\infty} \frac{1}{T-\tau} \int_0^{T-\tau} x(t) \cdot x(t+\tau) dt$$

Biased	Unbiased

3. **Compute and plot the autocorrelation of a square wave, sine wave and cosine wave**
   In case of a square wave the autocorrelation is a triangular wave, with an amplitude that is the double of the amplitude of the square wave. In case of having a sine wave, the autocorrelation is cosine and in case of a cosine the autocorrelation is still a cosine.

Square wave	Sine wave	Cosine wave

4. **Link between the autocorrelation and the RMS**
   The autocorrelation with a null phase displacement is equivalent to the RMS to the power of 2:

$$R_{xx}(0) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\cdot x(t+0)dt = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)^2 dt = RMS^2$$

5. **Definition of coherence**
   The coherence is defined as the normalisation of the cross spectrum. By a mathematical point of view, it can be defined as the ratio between the absolute value of the cross-spectrum to the power of two and the product between the two autospectrum:

$$\gamma_{AB}^2(f) = \frac{|S_{AB}^2(f)|}{S_{AA}(f)\cdot S_{BB}(f)}$$

   This value is always between 0 and 1 and it allows to underline that not always the components with maximum amplitude are the best, but the ones which, when taking the product, do not change by much their amplitude. If we compute the coherence for only one sample, it will always result equal to 1 since:

$$\gamma_{AB}^2(f) = \frac{\left|S_{AB}^2(f)\right|}{S_{AA}(f)\cdot S_{BB}(f)} = \frac{\left(S_{AB}(f)\right)^*\cdot S_{AB}(f)}{S_{AA}(f)\cdot S_{BB}(f)} = \frac{(A^*(f)\cdot B(f))^*\cdot A^*(f)\cdot B(f)}{A^*(f)\cdot A(f)\cdot B^*(f)\cdot B(f)} = \frac{B^*(f)\cdot A(f)\cdot A^*(f)\cdot B(f)}{A^*(f)\cdot A(f)\cdot B^*(f)\cdot B(f)} = 1$$

6. **Compare the spectrum of an ideal and real impulse hammer by drawing qualitatively the spectrums, explain the reason of their difference and explain which parameter effect the spectrum of an impulse hammer**
   In case of an ideal impulse hammer the spectrum is a flat value whereas in the real case the spectrum is a line that decreases its amplitude with the increase of the frequency. Of course, between the ideal and the real cases, also the function in the time domain changes. In fact, in the ideal case we are exciting infinite frequencies which means give infinite energy to the system, whereas in the real case this doesn't happen because the energy that is given to the structure is finite. If the point is hard the hammer is not like a low pass filter because I'm exciting the high frequencies of the system, on the other hand if the point is soft the hammer works like a pass low filter because it excites mainly the low frequencies.

Ideal impulse	Real impulse

7. **Define and compare the excitation techniques to measure the FRF of a mechanical system. Please answer with a table**

Type of technique	Definition	Pros	cons
Step sine technique	In this technique we excite our mechanical system with a harmonic wave. Since we are interested in a certain range of frequencies, we need to change the frequency of the impute force. To do that usually an exponential force is chosen. The frequency of the input is modified discretely, often by imposing a frequency step to the increment. It's really important to set properly the step otherwise we could miss some frequencies of interest	• We are in the steady state condition • It is accurate because we everything to be steady	• Problem of the resolution (since we are using a discrete frequency step) • Very time consuming • Expensive due to the expensive devices and equipment required
Sweep sine technique	In this technique we excite our mechanical system with a sweep sine which is a function that keeps on decreasing its period. We overcome the problem of the discrete step that we had in the step sine approach.	• no problem on frequency resolution since I excite all the frequencies • faster than the step sine	• we are never in the steady state condition • FRF actually never reached because we keep on increasing the frequency (this can be reduced by setting a very low rate of increase) • Expensive due to the expensive devices and equipment required
Impulsive technique	In this technique we excite our mechanical system by hitting the system with an impulse that is characterised by a very short duration.	• it's the fastest • It is cheap	• It is transient and not steady state at all and so FRF is never reached • Maybe difficult to have an adequate energy input without damaging the system • We could have some problem in case of overdamped system • Difficult to reaper and so we need for sure to make an average
White noise technique	In this technique we excite our mechanical system with a shaker fed with white noise signal	• Easy to implement	• It is transient and not steady state at all and so FRF is never reached • Could not cover the frequency of interest because it's a random signal • Expensive due to the equipment required