

# Hydraulic Similarity

## Exercise 1.

A centrifugal pump running at  $1450rpm$  provides a volumetric flow rate  $\dot{V}_d = 0.153m^3/s$ , a head  $H_d = 39m$  and requires a mechanical power equal to  $\dot{W}_d = 68kW$  at peak efficiency. It is required to determine the new peak efficiency point when:

- a) The angular speed is  $1200rpm$ , assuming constant diameter
- b) The diameter reduces from  $300mm$  to  $200mm$ , keeping constant the rotational speed at  $1450rpm$ .

## Data:

$rpm_d = 1450$	$\dot{V}_d = 0.153 \text{ m}^3/s$	$H_d = 39 \text{ m}$	$D_A = 0.3 \text{ m}$
$rpm_a = 1200$	$\dot{V}_a = ?$	$H_a = ?$	$D_a = D_d = 0.3 \text{ m}$
$rpm_b = 1450$	$\dot{V}_b = ?$	$H_b = ?$	$D_b = 0.2 \text{ m}$

## Calculation:

Since, in our analysis, we want to conserve the efficiency, we are forced to work under similarity conditions. So, when comparing the design case (d) with the two new cases (a) and (b) we first have to assure that the non-dimensional parameters of flow rate and  $\Phi$  and head ( $\Psi$ ) are conserved. Furthermore, in case (a) we conserved the diameter, while in case (b) we conserve the rotational speed.

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a)

$$\dot{V}_a = \dot{V}_d \frac{n_a}{n_d} = 0.127 \text{ m}^3/\text{s}$$

$$H_a = H_d \left( \frac{n_a}{n_d} \right)^2 = 26.7 \text{ m}$$

b)

$$\dot{V}_b = \dot{V}_d \left( \frac{D_b}{D_d} \right)^3 = 0.0453 \text{ m}^3/\text{s}$$

$$H_b = H_d \left( \frac{D_b}{D_d} \right)^2 = 17.33 \text{ m}$$

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### Exercise 3.

A hydraulic turbine has to be designed to be installed in a plant characterized by a head of  $1200m$ , with an available flow rate of  $4m^3/s$ . The statistical diagrams (Baljé diagrams) indicates that the most suitable machine (namely, the machine that guarantees the maximum performance) for such an application is the Pelton turbine. The non-dimensional parameters of the Pelton chosen for the design are  $\omega_s = 0.093$  and  $D_s = 16$ ; for this operating point the statistical diagrams provide an estimated efficiency of 0.88. Prior to construct and install the machine, a preliminary experimental validation of the desing is required; in the laboratory context we have an available head of  $200m$  and a power for the brake of  $15kW$ . It is required to compute the rotational speed and the diameter of the real turbine and the ones of the model turbine tested in the laboratory.

### Data:

<u>desing turbine</u>		<u>laboratory turbine</u>
$H_A = 1200 \text{ m}$	$\dot{V}_A = 4 \text{ m}^3/s$	$H_L = 200 \text{ m}$
$\eta = 0.88$	$D_{sA} = 16$	$\dot{V}_L = 15 \text{ kW}$

### Calculation:

By using the non-dimensional parameters assigned in the design phase we can compute easily the real rotational speed and the diameter, once we have converted the head from meters (typical unit used in hydraulics) to energetic units ( $J/kg$ ) by multiplying by the gravitational acceleration  $g$ :

$$\omega_s = \omega_A \frac{\sqrt{\dot{V}_A}}{(gH_A)^{3/4}} \Rightarrow \omega_A = \omega_s \frac{(gH_A)^{3/4}}{\sqrt{\dot{V}_A}} = 52.5 \text{ rad/s}$$
$$D_s = D_A \frac{(gH_A)^{1/4}}{\sqrt{\dot{V}_A}} \Rightarrow D_A = D_s \frac{\sqrt{\dot{V}_A}}{(gH_A)^{1/4}} = 3.07 \text{ m}$$

To evaluate now the machine model in the laboratory context, we have first to compute the flow rate available in the lab. With such an information, we

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can compute the angular speed and the diameter as we have just done for the design configuration. The turbine power can be expressed as

$$\dot{W}_{turbina} = \dot{m} \cdot \Delta h = \rho \dot{V} \cdot gH \cdot \eta$$

Since we want to conserve the hydraulic similarity between the real context and the laboratory test we can assume that the efficiency is the same:  $\eta_A = \eta_L = \eta = 0.88$

From this condition we can compute the maximum flow rate available in the laboratory tests.

$$\dot{V}_L = \frac{\dot{W}_L}{\eta \rho \cdot gH_L} = 0.0087 \text{ m}^3/s$$

Now, from the flow rate and the head in the laboratory tests, we can compute the angular speed and the machine diameter to respect the similarity with the design machine:

$$\omega_s = \omega_L \frac{\sqrt{\dot{V}_L}}{(gH_L)^{3/4}} \Rightarrow \omega_L = \omega_s \frac{\sqrt{(gH_L)^{3/4}}}{\dot{V}_L} = 293.4 \text{ rad/s} \approx 2800 \text{ rpm}$$

$$D_s = D_L \frac{(gH_L)^{1/4}}{\sqrt{\dot{V}_L}} \Rightarrow D_L = D_s \frac{\sqrt{\dot{V}_L}}{(gH_L)^{1/4}} = 0.224 \text{ m}$$

## Conservazione dell'energia ed equazione di eulero

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### Esercizio 4.

Un serbatoio è posto ad una quota di 25 metri dal suolo e ad una pressione pari a quella atmosferica.

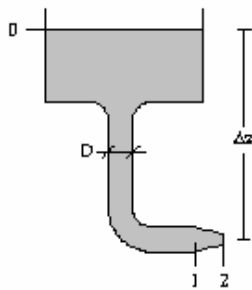
In assenza di perdite si calcoli la portata di acqua fluente da un condotto con diametro  $D = 200 \text{ mm}$  e lunghezza  $L = 50 \text{ m}$ , che termina con un ugello di diametro  $d = 80 \text{ mm}$  posto a livello del suolo che scarica in ambiente atmosferico. Si determini inoltre la pressione nel tubo immediatamente prima dell'ugello.

a) Supponendo ora le seguenti fonti di perdite localizzate: 5 curve a  $90^\circ$  (ciascuna con coefficiente di perdita localizzato  $\xi = 0.3$ ; ( $\xi = n^\circ$  quote cinetiche perse), valvola di aspirazione con filtro ( $\xi = 0.8$ ) ed il coefficiente di perdita per attrito nel tubo  $\lambda = 0.01$ , si calcoli il diametro necessario al bocchaglio per ottenere la stessa portata calcolata in precedenza.

(Valutazione delle perdite di carico:  $Y_{distr.} = L/D \cdot \lambda \cdot \frac{V^2}{2g}$ ;  $Y_{loc.} = \sum \xi \cdot \frac{V^2}{2g}$ )

b) Si calcoli il diametro al bocchaglio necessario per ottenere la stessa portata calcolata in precedenza con una condotta di diametro  $D'_1 = 150 \text{ mm}$ .

### Dati:



$$\begin{array}{lll} z_0 = 25 \text{ m} & z_1 = 0 \text{ m} & z_2 = 0 \text{ m} \\ L_{tubo} = 50 \text{ m} & D_1 = 0.2 \text{ m} & D_2 = 0.08 \text{ m} \\ \xi = 0.3 & \lambda = 0.01 & p_1 = p_2 = p_{atm} \end{array}$$

### Svolgimento:

$S_1$  = sezione del condotto

$S_2$  = sezione di sbocco dell'ugello

$$S_1 = \pi \cdot \frac{D_1^2}{4} = 0.0314 \text{ m}^2$$

$$S_2 = \pi \cdot \frac{D_2^2}{4} = 0.005 \text{ m}^2$$

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$$V_2 = \sqrt{2g \cdot z_0} = 22.15 \text{ m/s}$$

$$\dot{Q} = V_2 \cdot S_2 = 0.111 \text{ m}^3/\text{s}$$

$$V_1 = \frac{\dot{Q}}{S_1} = 3.54 \text{ m/s}$$

Calcoliamo a questo punto la pressione nella sezione (1) scrivendo la conservazione dell'en. meccanica tra le due sezioni 0, 1 in assenza di  $l_w$  e  $l$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_0}{\rho} + \frac{V_0^2}{2} + gz_0$$

con  $V_0 = 0$  otteniamo

$$p_1 = \rho \cdot \left( gz_0 - \frac{V_1^2}{2} \right) = 2.39 \text{ bar}_A$$

**a)** Calcoliamo come suggerito dal testo dell'esercizio le perdite localizzate e

distribuite

*perdite localizzate*

$$Y_{loc.} = \sum \xi \cdot \frac{V_1^2}{2g} = 2.3 \frac{V_1^2}{2g}$$

*perdite distribuite*

$$Y_{distr.} = \frac{L_{tubo}}{D_1} \cdot \lambda \cdot \frac{V_1^2}{2g} = 2.5 \frac{V_1^2}{2g}$$

ricaviamo la perdita totale come somma delle due

$$Y_{tot} = Y_{loc.} + Y_{distr.} = 4.8 \frac{V_1^2}{2g} = 3.066 \text{ m}$$

Riscriviamo la conservazione dell'en. meccanica tenendo in conto delle perdite tra le sezioni (0) e (2)

$$\frac{p_0}{\rho} + gz_0 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gY_{tot}$$

abbiamo  $p_0 = p_2 = p_{amb}$  e quindi

$$V_2 = \sqrt{2g \cdot (z_0 - Y_{tot})} = 20.8 \text{ m/s}$$

## Conservazione dell'energia ed equazione di eulero

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imponendo la portata del caso precedente, la sezione di scarico sarà

$$S_2 = \frac{\dot{Q}}{V_2} = 0.00536 \text{ m}^2$$

con un diametro

$$D_2 = \sqrt{\frac{4 \cdot S_2}{\pi}} = 0.0826 \text{ m}$$

b) Cambiando la sezione del condotto va ricalcolata la  $V_1$  e di conseguenza le perdite

$$S'_1 = \pi \cdot \frac{D'^2_1}{4} = 0.0177 \text{ m}^2$$

$$V'_1 = \frac{\dot{Q}}{S'_1} = 6.28 \text{ m/s}$$

*perdite localizzate*

$$Y_{loc.} = \sum \xi \cdot \frac{V'^2_1}{2g} = 2.3 \frac{V'^2_1}{2g}$$

*perdite distribuite*

$$Y'_{distr.} = \frac{L_{tubo}}{D'_1} \cdot \lambda \cdot \frac{V'^2_1}{2g} = 3.33 \frac{V'^2_1}{2g}$$

$$Y'_{tot} = Y_{loc.} + Y_{distr.} = 5.63 \frac{V'^2_1}{2g} = 11.32 \text{ m}$$

il bilancio energetico tra le due sezioni diventa:

$$\frac{p_0}{\rho} + gz_0 = \frac{p_2}{\rho} + \frac{V'^2_2}{2} + gY'_{tot}$$

abbiamo  $p_0 = p_2 = p_{amb}$  e quindi

$$V'_2 = \sqrt{2g \cdot (z_0 - Y'_{tot})} = 16.38 \text{ m/s}$$

$$S'_2 = \frac{\dot{Q}}{V'_2} = 0.00677 \text{ m}^2$$

con un diametro

$$D'_2 = \sqrt{\frac{4 \cdot S'_2}{\pi}} = 0.093 \text{ m}$$