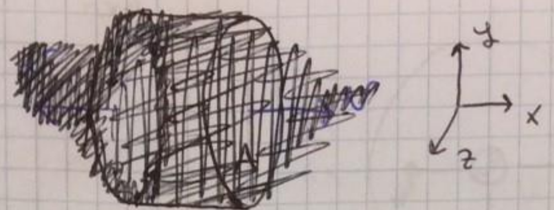
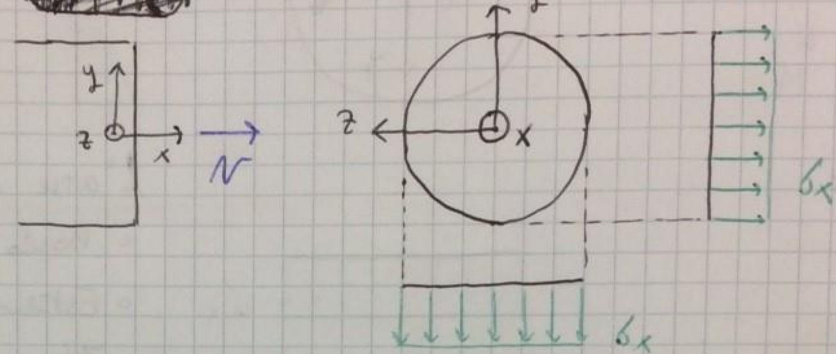
Introduzione

DE SAINT VENANT

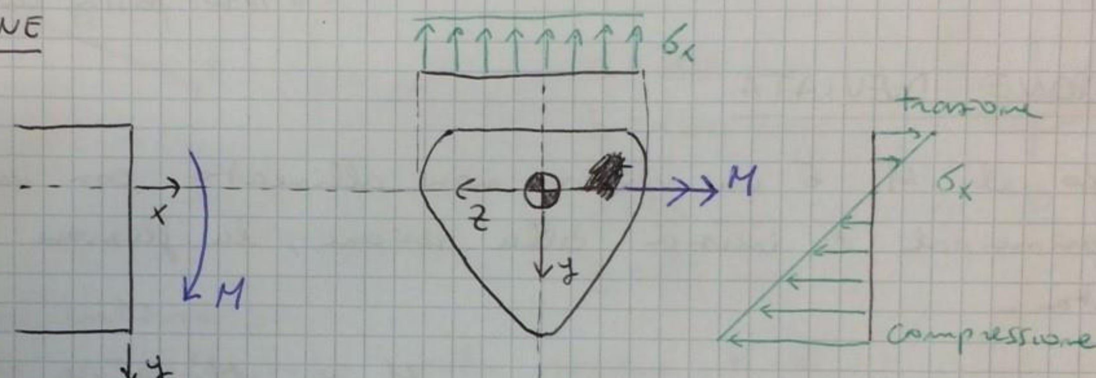
TRAZIONE



$$\sigma_x = \frac{N}{A}$$



FLESSIONE



Se sulla sezione agisce solo M allora l'asse neutro passa per il baricentro

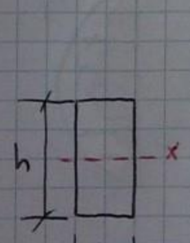
$$\sigma_x = E\,\varepsilon_x = -E\,\Gamma\, y$$

$$\Gamma = \frac{1}{\rho} = -\frac{M}{E J_z} \qquad \Rightarrow \qquad \sigma_x = -E\Gamma y = \frac{M y}{J_z}$$

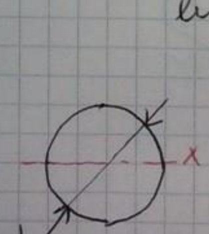
Formula di Eulero-Bernoulli

Formula di Navier

Gli sforzi si distribuiscono linearmente lungo y

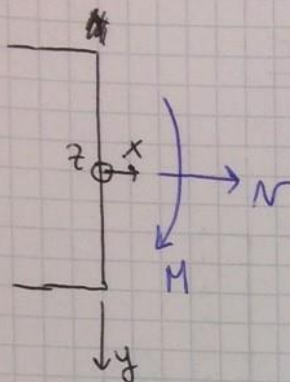


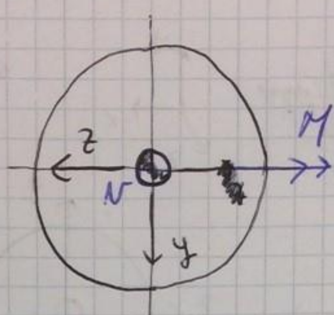
$$J_x = \frac{b h^3}{12}$$

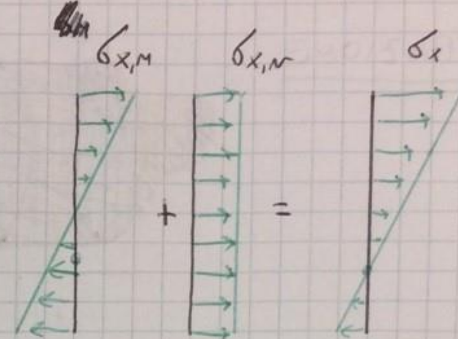


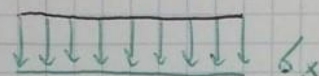
$$J_x = \frac{\pi d^4}{64}$$

FLESSIONE + TRAZIONE









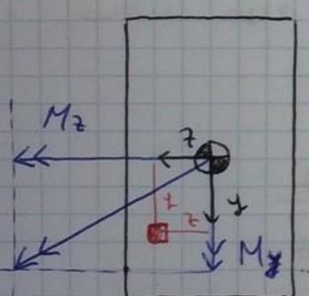
L'asse neutro può essere

- Bordo della sezione
- Esterno alla sezione
- Non passa dal baricentro

FLESSIONE DEVIATA

Quando il M è un vettore non allineato con uno degli assi principali di inerzia della sezione, la flessione si dice deviata.

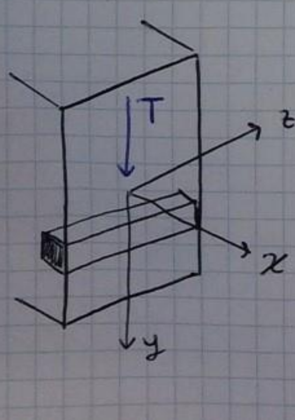
$$\sigma_x(z,y) = \frac{M_z \cdot y}{J_z} - \frac{M_y \cdot z}{J_y}$$

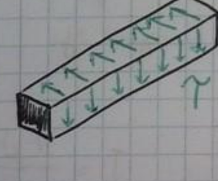


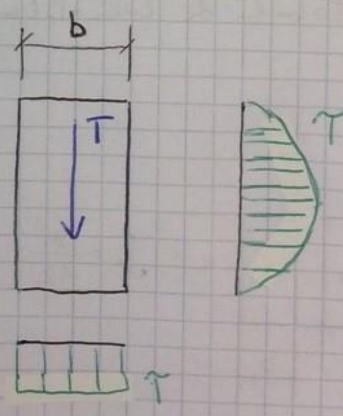
Posizione asse neutro:

$$\sigma_x(z,y) = 0 = \frac{M_z\, y}{J_z} - \frac{M_y \cdot z}{J_y}$$

TAGLIO



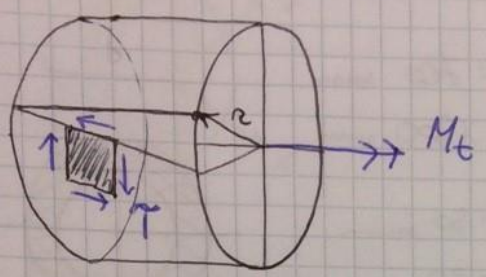


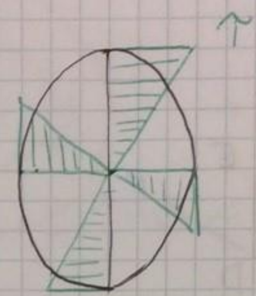


$$\tau = \frac{T\, s(y)}{J\, b} = \frac{T}{2J}\left(\frac{h^2}{4} - y_1^2\right)$$

Per sezioni rettangolari: $\tau_{max} = \frac{3}{2}\frac{T}{A}$ Per sezioni circolari $\tau_{max} = \frac{4}{3}\frac{T}{A}$

TORSIONE

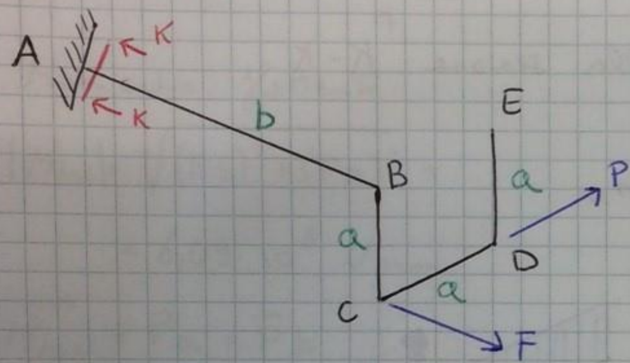




$$\tau = \frac{M_t \cdot r}{J_P}$$

sezione circolare:

$$J_P = \frac{\pi d^4}{32}$$



P = 4000 N

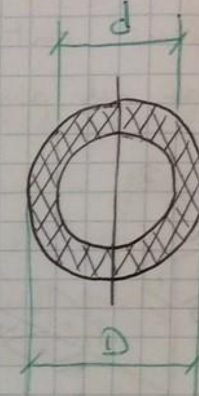
F = 1500 N

a = 700 mm

b = 1500 mm

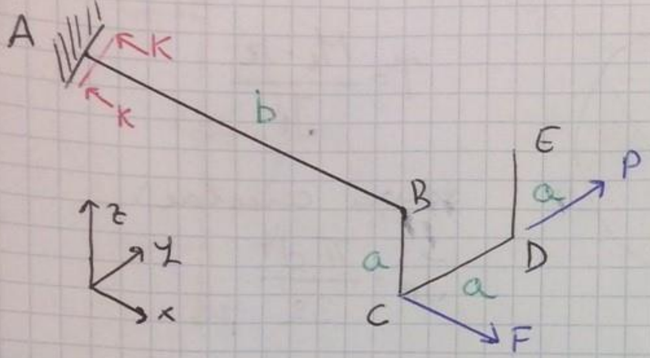
d = 50 mm

D = 80 mm



- Azioni interne
- Punto più sollecitato della sezione K-K

Tema d'esame 03/03/2015



$P = 4000\ N$

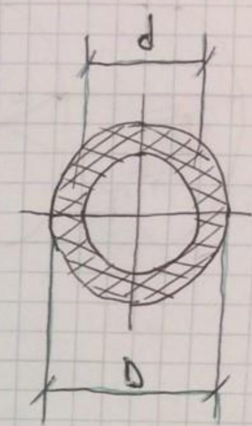
$F = 1500\ N$

$a = 700\ mm$

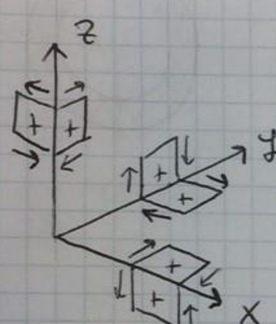
$b = 1500\ mm$

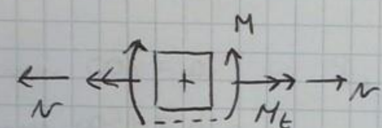
$d = 50\ mm$

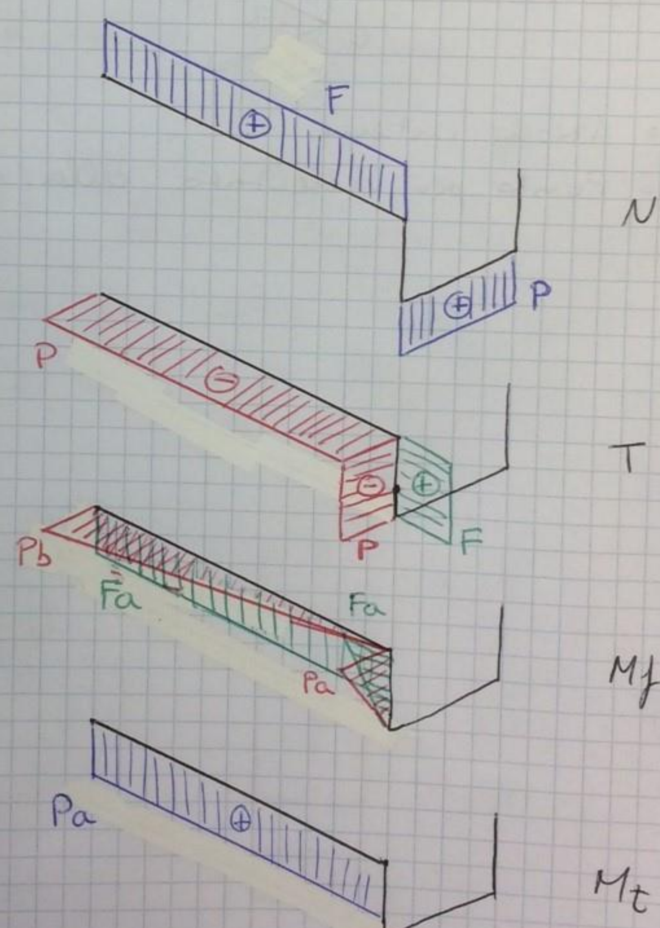
$D = 80\ mm$



- Azioni interne
- Sforzi nel punto più sollecitato della sezione K-K

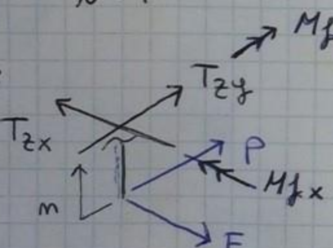






$\overline{CD}$: $N = P$

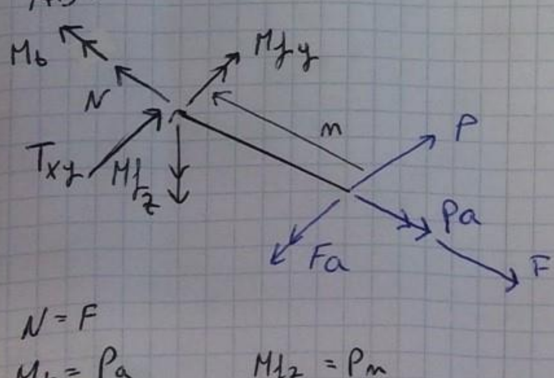
$\overline{BC}$:



$T_{zx} = F \qquad Mf_x = P\,m$

$T_{zy} = -P \qquad Mf_y = F\,m$

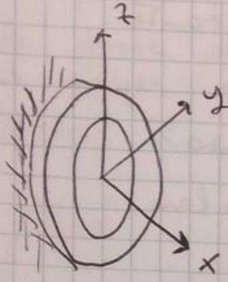
$\overline{AB}$:

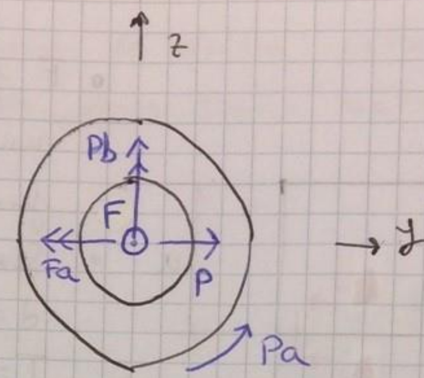


$N = F$

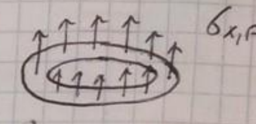
$M_t = Pa \qquad Mf_z = Pm$

$T_{xy} = -P \qquad Mf_y = Fa$



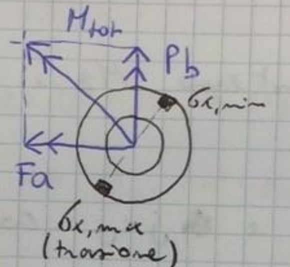


Azione assiale



$$\sigma_{x,F} = \frac{F}{A} = \frac{1500\ N}{\frac{\pi(D^2-d^2)}{4}} = \frac{1500\ N}{3{,}06 \cdot 10^3\ mm^2} = 0{,}49\ MPa$$

Momento flettente



$$|M_{tot}| = \sqrt{(P \cdot b)^2 + (Fa)^2} =$$

$$= 6{,}09 \cdot 10^6\ Nmm$$

$$\overrightarrow{M_{tot}} = \vec{F}a + \vec{P}b$$

Posso "comporre" i due momenti flettenti poiché la sezione è assialsimmetrica e J = costante

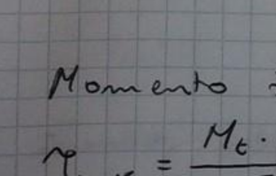
$$\sigma_x = \frac{M_{ftot} \cdot D/2}{J_{\oplus}} = \frac{M_{ftot} \cdot D/2}{\frac{\pi(D^4-d^4)}{64}} = \frac{6{,}09 \cdot 10^6\ Nmm \cdot 40\ mm}{1{,}704 \cdot 10^6\ mm^4} = 143\ MPa$$

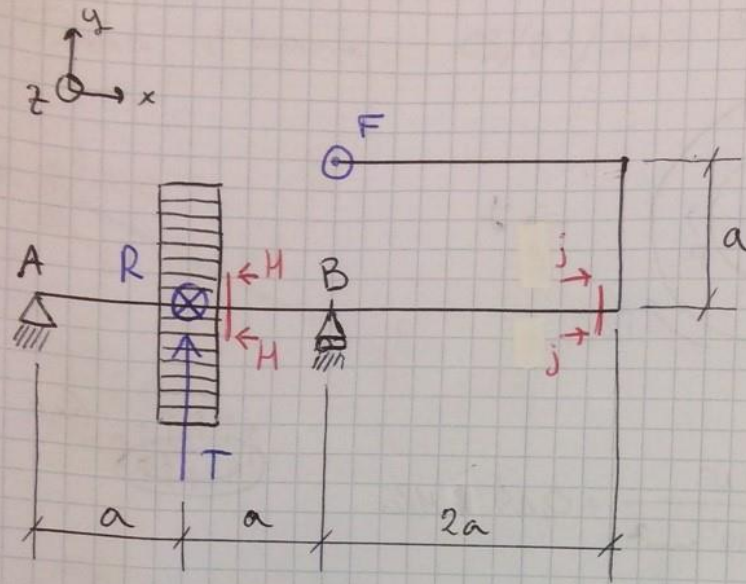
Taglio

$$\tau = \frac{4}{3}\frac{T}{A} = \frac{4}{3}\frac{4000\ N}{3{,}06 \cdot 10^3\ mm^2} =$$

Momento torcente

$$\tau_{max} = \frac{M_t \cdot D/2}{J_P} = \frac{(4000 \cdot 200)\ Nmm \cdot 40\ mm}{\frac{\pi(D^4-d^4)}{32} = 3{,}407 \cdot 10^6\ mm^4} = 32{,}9\ MPa$$





- F forza rotante "resistente" a regime di funzionamento
- Sezione circolare

$d = 30$ mm

$T = 1000$ N

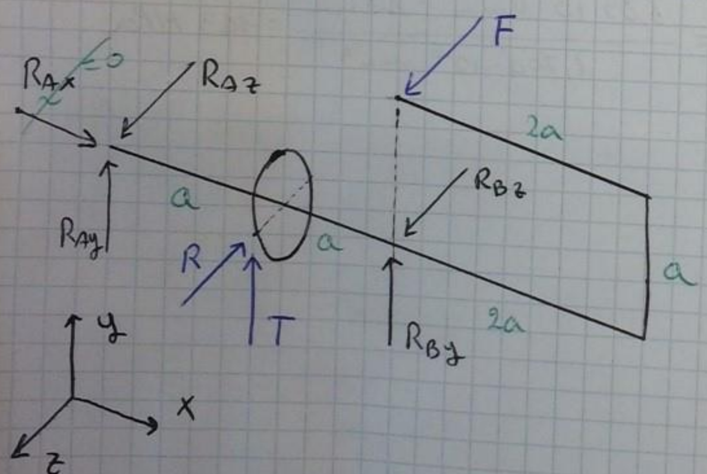
$R = 0{,}3 \cdot T$

$a = 300$ mm

Diametro ruota dentata $d_R = 600$ mm

→ Calcolo azioni interne M_f e M_t separate in base alle azioni di spinta

→ Sforzi in H-H e j-j commentando le loro variazioni nel tempo.



Osservazioni:

- R e T sono due forze fisse rispetto al sistema di riferimento x, y, z.
- F è solidale all'albero e ruota con esso.

Possiamo innanzitutto utilizzare l'equilibrio dei Momenti intorno all'asse (x) dell'albero che permette di calcolare F senza problemi dal punto di vista di forze fisse/rotanti:

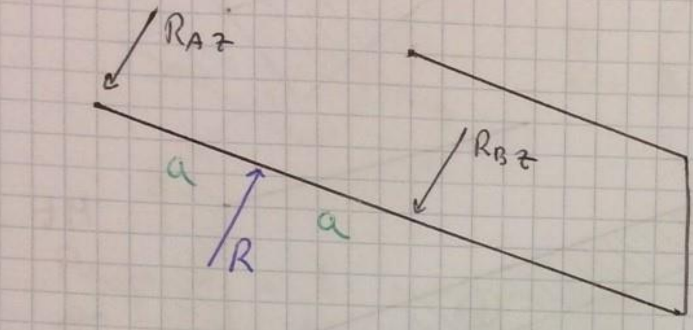
$T = 1000 \text{ N} \quad \rightarrow \quad R = 0{,}3 \cdot T = 300 \text{ N}$

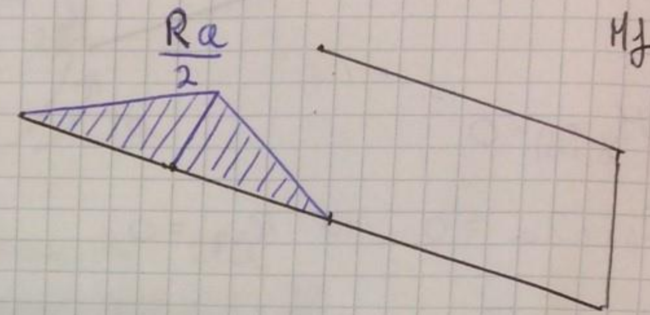
$$\sum M_{(x)} = 0 \qquad T\frac{d_R}{2} - Fa = 0 \rightarrow F = T\frac{d_R}{2}\frac{1}{a} = 1000\text{ N}\frac{600\text{ mm}}{2 \cdot 300\text{ mm}} = 1000 \text{ N}$$

$$\sum F_x = 0 \qquad R_{Ax} = 0$$

È importante scomporre le azioni interne in base alle forze poichè alcune forze sono rotanti e altre fisse.

Forza R





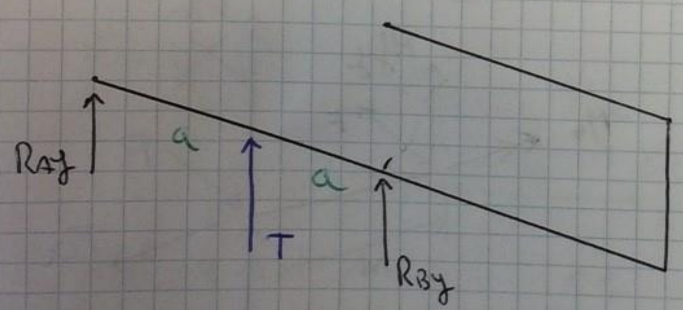
Per simmetria $R_{Az} = R_{Bz} = \frac{R}{2} = 150\ N$

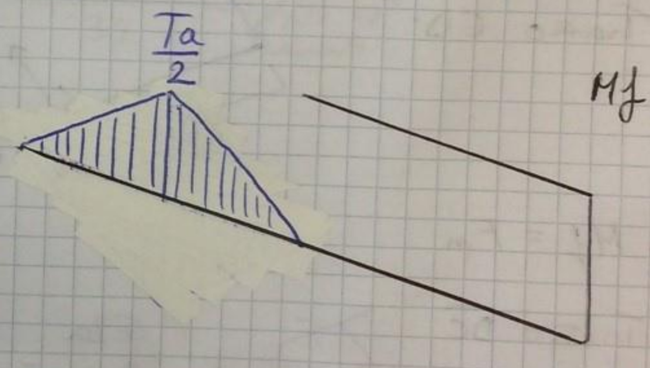
Oppure: $\Sigma M_{(y)} = 0 \quad R\cancel{a} - R_{Bz} 2\cancel{a} = 0 \rightarrow R_{Bz} = \frac{1}{2} R = 150\ N$

$\Sigma F_z = 0 \rightarrow \quad R_{Az} + R_{Bz} - R = 0 \longrightarrow R_{Az} = R - R_{Bz} = \frac{1}{2} R = 150\ N$

R fissa $\Rightarrow$ R_{Az} e R_{Bz} fissi

Forza T

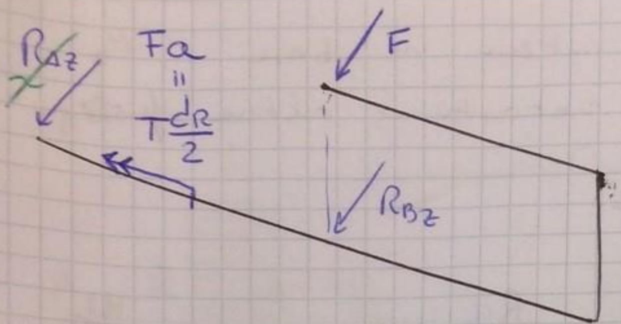


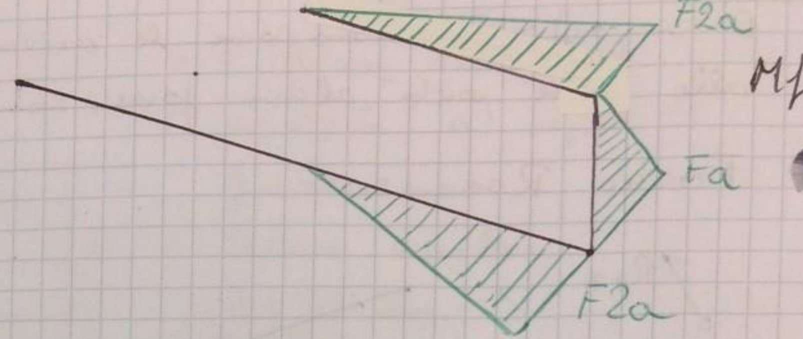


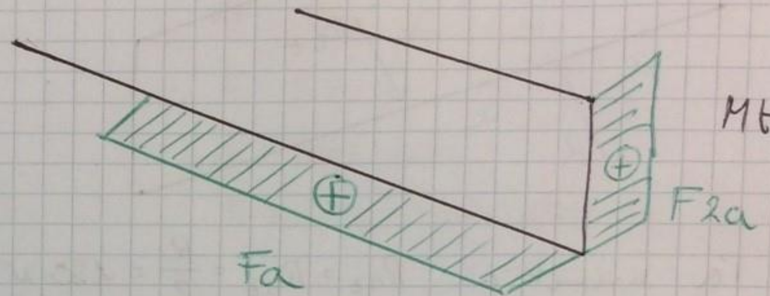
Per simmetria $R_{Ay} = R_{By} = -\frac{T}{2}$

$\Sigma M_A = 0 \quad T\cancel{a} + 2\cancel{a} R_{By} = 0 \quad R_{By} = -\frac{T}{2}$

$T + R_{Ay} + R_{By} = 0 \qquad R_{Ay} = -T - R_{By} = -T + \frac{T}{2} = -\frac{T}{2}$



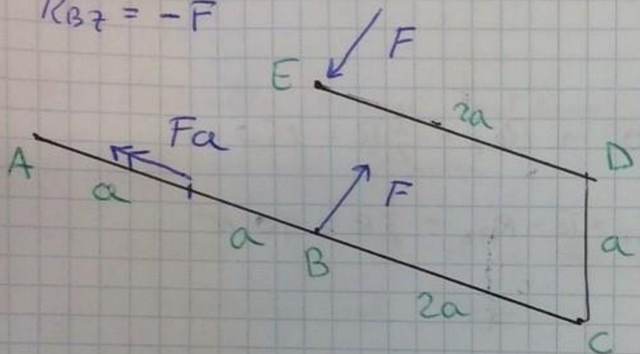




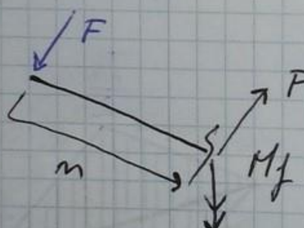
$$\Sigma M_{y\,(B)} = 0$$

$$R_{Az} \cdot 2a = 0 \Rightarrow R_{Az} = 0$$

$$R_{Bz} = -F$$

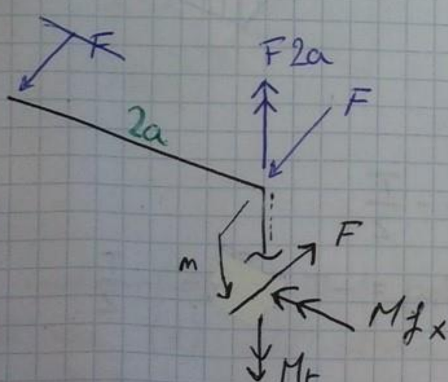


Tramo ED:



$$M_f = Fm$$

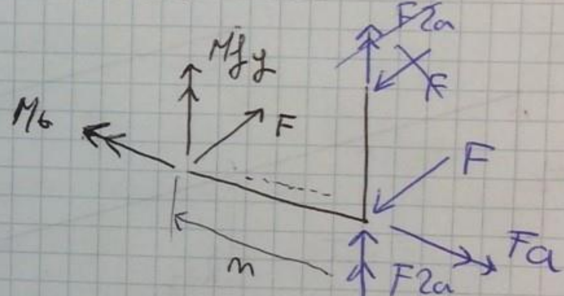
Tramo $\overline{DC}$



$$M_t = 2aF$$

$$M_{f_x} = Fm$$

Tramo $\overline{BC}$



$$M_t = Fa$$

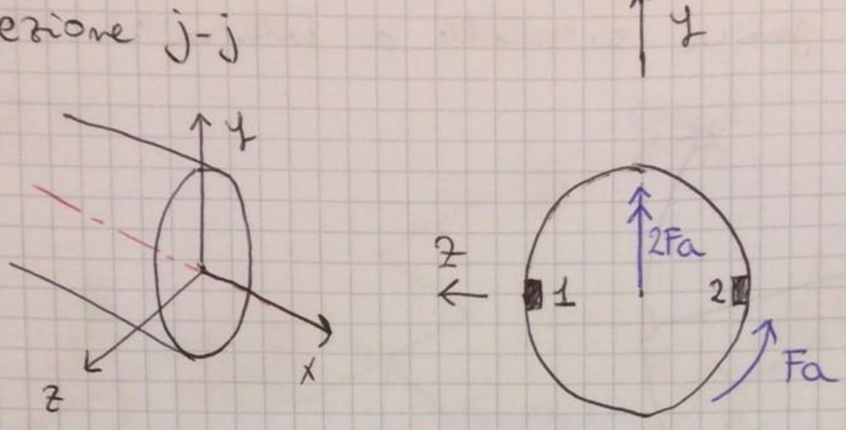
$$M_{f_y} - Fm + F2a = 0$$

$$M_{f_y} = Fm - F \cdot 2a$$

$m = 0 \quad M_{f_y} = -F2a$

$m = 2a \quad M_{f_y} = 0$

Sezione j-j



Punto 1 e 2 maggiormente sollecitati da 2Fa

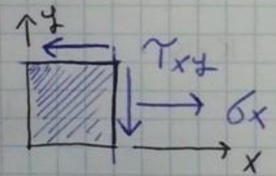
$$\sigma_x = \frac{M_f \cdot d/2}{J_\phi} = \frac{2Fa \cdot d/2}{\frac{\pi d^4}{64}} = \frac{2 \cdot 1000N \cdot 300\ mm \cdot \frac{30}{2} mm}{\underbrace{\frac{\pi\ 30^4\ mm^4}{64}}_{= 3{,}98 \cdot 10^4\ mm^4}} = 226{,}35\ MPa$$

Punto 1 : $\sigma_x = 226{,}35\ MPa$

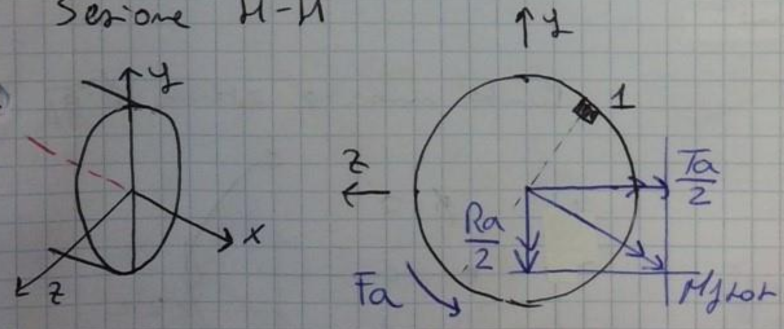
Punto 2 : $\sigma_x = -226{,}35\ MPa$

$$\tau_{max} = \frac{M_t \cdot d/2}{J_P} = \frac{Fa \cdot \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{1000N \cdot 300\ mm \cdot 30\ mm/2}{\underbrace{\frac{\pi\ 30^4\ mm^4}{32}}_{= 7{,}95 \cdot 10^4\ mm^4}} = 56{,}59\ MPa$$

Punto 1:



Sezione H-H



$\frac{Ta}{2}$ e $\frac{Ra}{2}$ sono fissi rispetto a un sistema di riferimento esterno, ma l'albero gira!

$$\vec{M}_{f\,tot} = \frac{\vec{Ta}}{2} + \frac{\vec{Ra}}{2} \Rightarrow |M_{f\,tot}| = \sqrt{\left(\frac{Ta}{2}\right)^2 + \left(\frac{Ra}{2}\right)^2} = 1{,}566 \cdot 10^6\ N\,mm$$

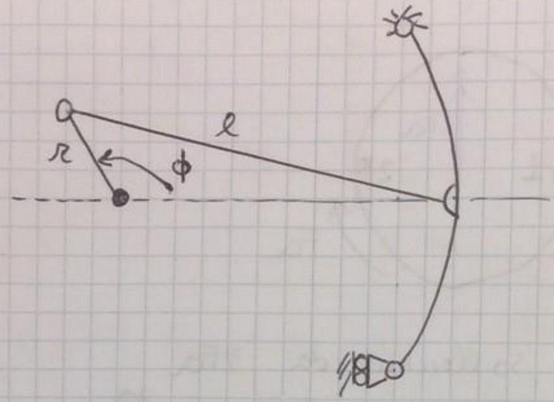
$$\sigma_x = \frac{M_{f\,tot} \cdot d/2}{J_{P\phi}} = \frac{1{,}566 \cdot 10^6\ Nmm \cdot 30\ mm/2}{\frac{\pi\ 30^4\ mm^4}{32}} = 59{,}08\ MPa$$

Punto 1

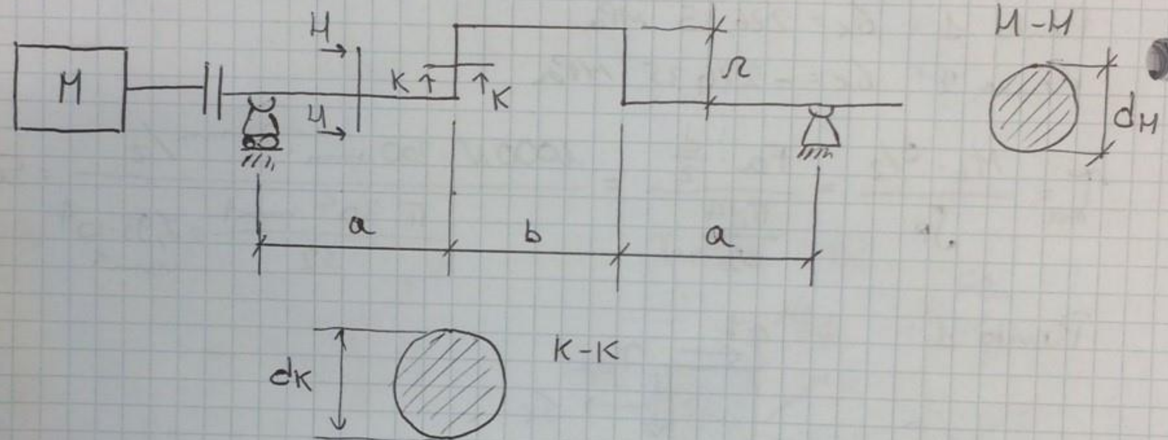
$\tau = 56{,}59\ MPa$ Punto 1: $\sigma_x(t) = \sigma_{x,a} \operatorname{sen}(\omega t) + \sigma_{x,m}$ $\begin{cases} \sigma_{x,m} = 0 \\ \sigma_{x,a} = 59{,}08\ MPa \end{cases}$

$\tau_{max}(t) = \tau_{max}$ $\begin{cases} \tau_a = 0 \\ \tau_m = 56{,}6\ MPa \end{cases}$

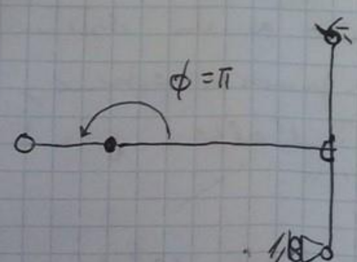
Macchina per prove a fatica su molle a balestra:



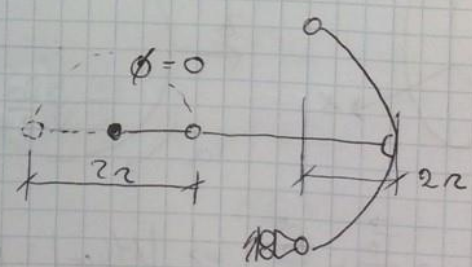
Albero di manovella:



Punto morto interno $\phi = \pi$



Punto morto esterno $\phi = 0$



$\lambda = \frac{r}{l} = 0{,}05$ $\approx$ biella di lunghezza infinita

$r = 25$ mm
$b = 50$ mm
$a = 70$ mm
$d_H = 15$ mm
$d_K = 20$ mm

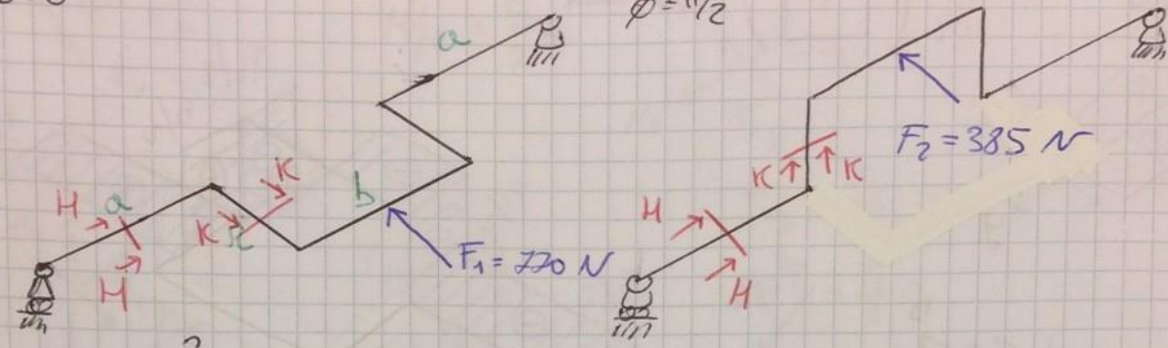
$\lambda = \frac{r}{l} \rightarrow l = \frac{r}{\lambda} = \frac{250 \text{ mm}}{0{,}05} = 500$ mm

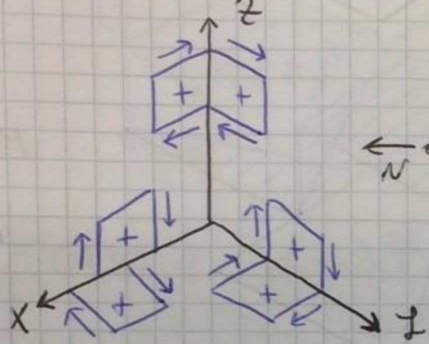
$\rightarrow$ Calcolare azioni interne per $\phi = 0$ e $\phi = \frac{\pi}{2}$ sull'albero di manovella e sforzi massimi sulle sezioni h-h e k-k

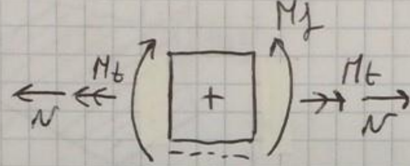
Due casi ~~...~~ in esame:

CASO 1
$\phi = 0$

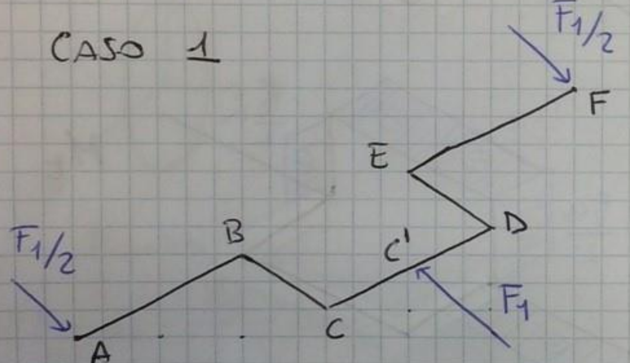
CASO 2
$\phi = \pi/2$

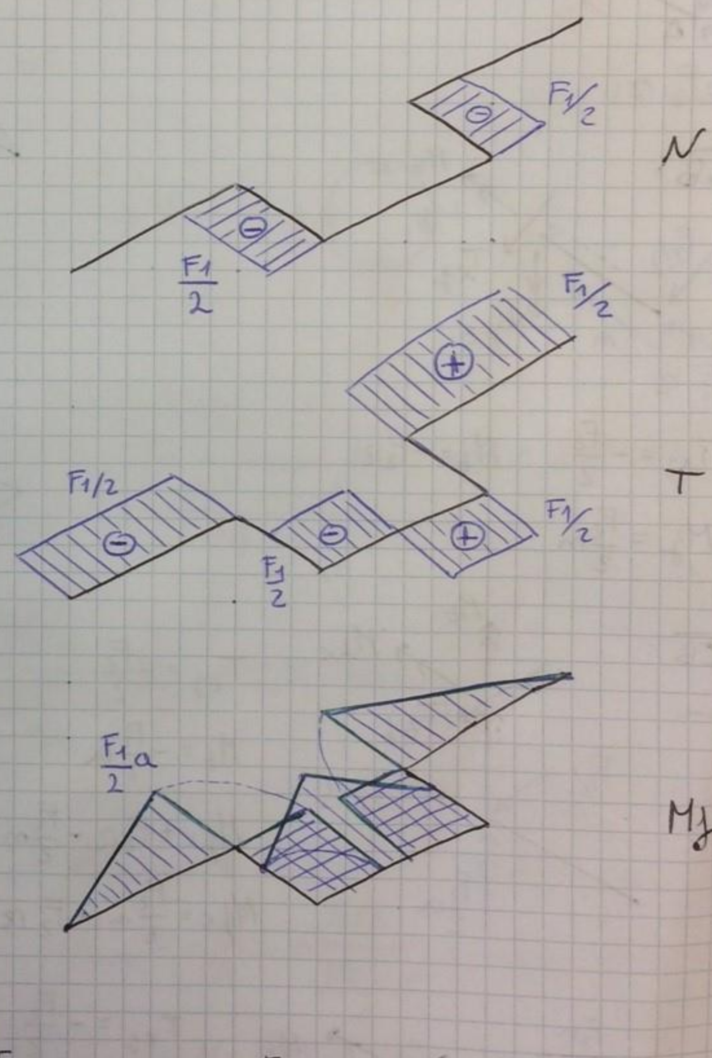




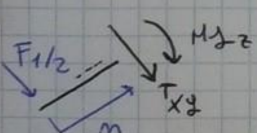


CASO $\underline{1}$





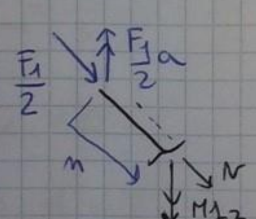
$\overline{AB}$:



$T_{xy} = -\frac{F_1}{2}$

$M_{fz} = \frac{F_1}{2} m$

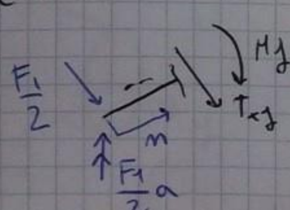
$\overline{BC}$:



$N = -\frac{F_1}{2}$

$M_{fz} = \frac{F_1 a}{2}$

$\overline{CC'}$:

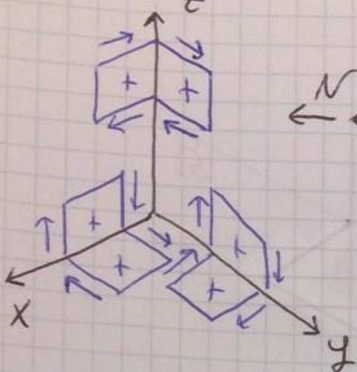


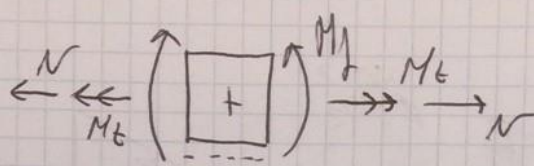
$T_{xy} = -\frac{F_1}{2}$

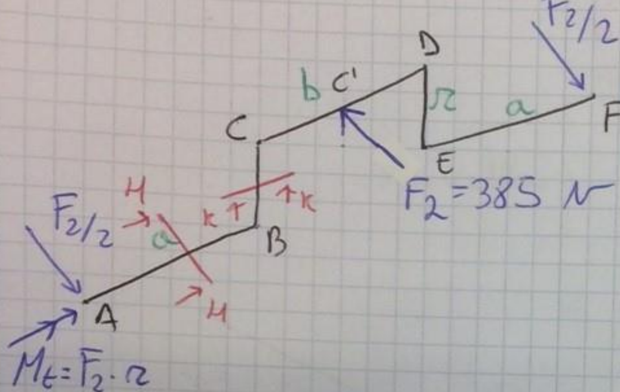
$M_{fz} - \frac{F_1}{2}a - \frac{F_1 m}{2} = 0 \quad M_{fz} = \frac{F_1}{2}a + \frac{F_1}{2}m$

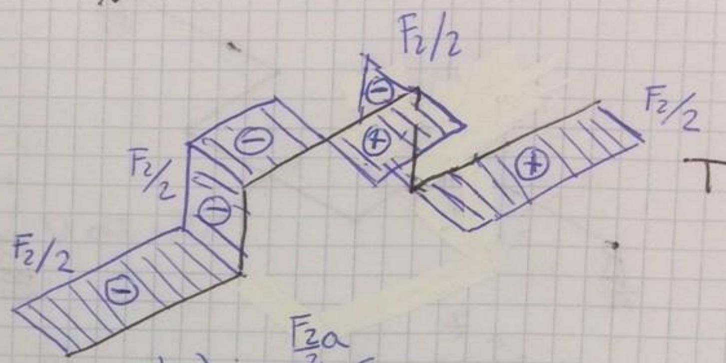
CASO 2

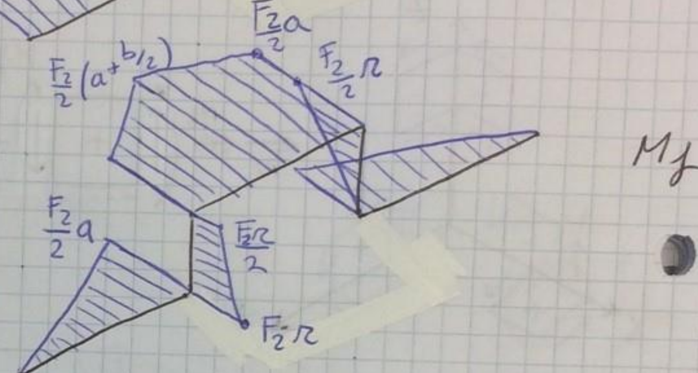
$\phi = \pi/2$

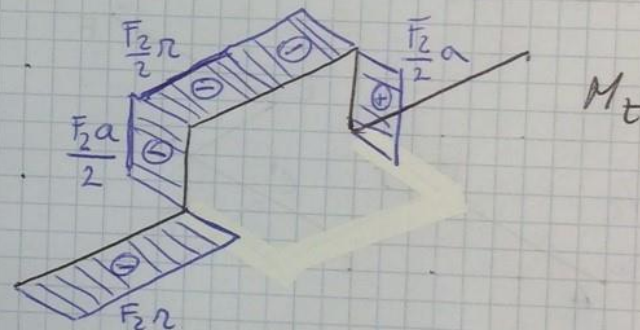




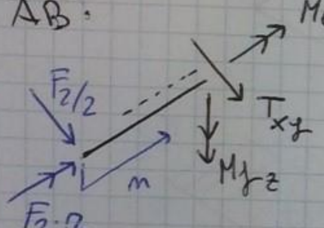








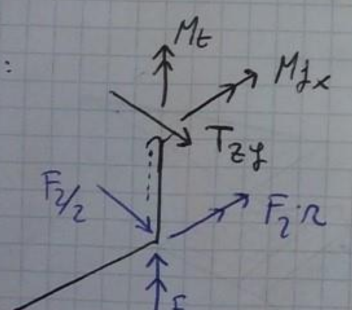
$\overline{AB}$:



$T_{xy} = -\frac{F_2}{2}$ $\qquad M_t = -F_2 r$

$M_f = \frac{F_2}{2} m$

$\overline{CB}$:



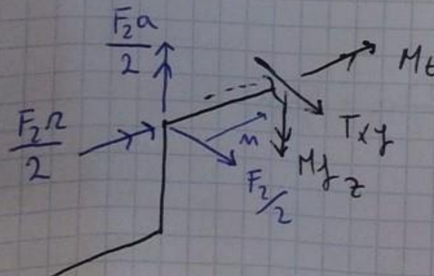
$T_{zy} = -\frac{F_2}{2}$

$M_t = -\frac{F_2}{2} a$

$M_{fx} + F_2 r - \frac{F_2}{2} m = 0$

$M_{fx} = \frac{F_2}{2} m - F_2 r$ $\qquad m = 0 \quad -F_2 r$

$m = r \quad -\frac{F_2}{2} r$



$T_{xy} = -\frac{F_2}{2}$

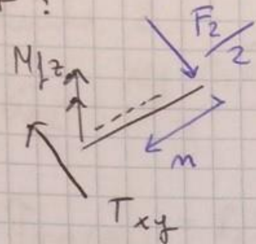
$M_t = -\frac{F_2}{2} r$

$M_{fz} - \frac{F_2}{2} a - \frac{F_2}{2} m = 0$

$M_{fz} = \frac{F_2}{2} a + \frac{F_2}{2} m$ $\qquad m = 0 \quad \frac{F_2}{2} a$

$m = b/2 \quad \frac{F_2}{2}(a + b/2)$

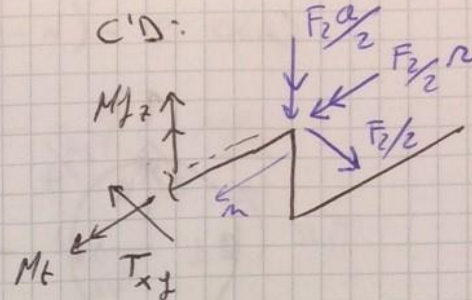
$\overline{EF}$:



$$T_{xy} = \frac{F_2}{2}$$

$$M_{fz} = \frac{F_2}{2} m$$

$\overline{C'D}$:

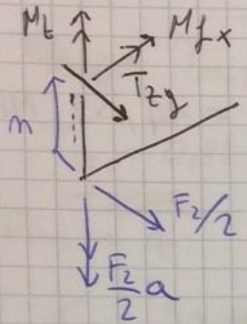


$$T_{xy} = \frac{F_2}{2}$$

$$M_t = -\frac{F_2}{2} r$$

$$M_{fz} = F_2 \frac{a}{2} + \frac{F_2}{2} m$$

$\overline{DE}$:

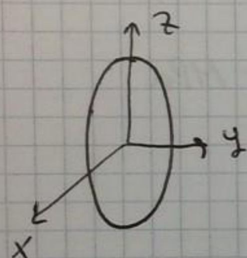


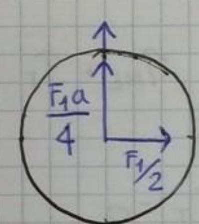
$$T_{zy} = -\frac{F_2}{2}$$

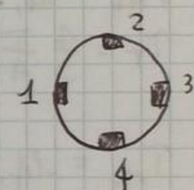
$$M_t = \frac{F_2}{2} a$$

$$M_{fx} = \frac{F_2}{2} m$$

CASO 1 - (h-h)







$d_h = 15$ mm

Punto 1

$$\sigma_x = \frac{M_f \cdot d_h/2}{J} = \frac{\frac{F_1 a}{4} \cdot d_h/2}{\frac{\pi d_h^4}{64}} = \frac{\frac{220 N \cdot 20 mm}{4} \cdot \frac{15 mm}{2}}{\frac{\pi 15^4 mm^4}{64}} = 40,67 \text{ MPa}$$

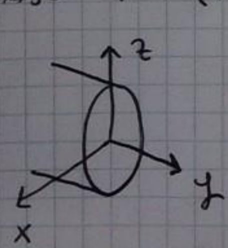
$\tau = 0$

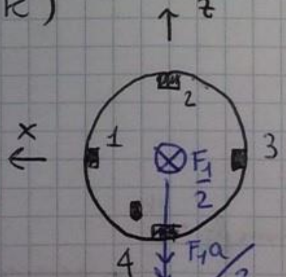
Punto 2

$$\tau_{max} = \frac{4}{3}\frac{T}{A} = \frac{4}{3}\frac{F_1/2}{\frac{\pi d_h^2}{4}} = \frac{4}{3}\frac{220N/2}{\frac{\pi 15^2 mm^2}{4}} = 2,9 \text{ MPa}$$

$\sigma_x = 0$

CASO 1 - (k-k)





Punto 3:

$d_k = 20$ mm

$$\sigma_{x,N} = -\frac{N}{A} = -\frac{F_1/2}{\frac{\pi 20^2 mm^2}{4}} = -1,26 \text{ MPa}$$

$$\sigma_{x,Mf} = \frac{M_f\, d_k/2}{J} = \frac{\frac{F_1 a}{2} \cdot \frac{d_k}{2}}{\frac{\pi d_k^2}{64}} = 34,31 \text{ MPa}$$

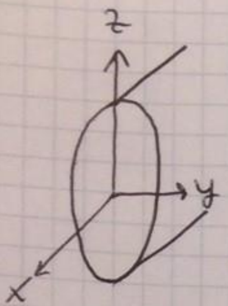
$$\sigma_x = \sigma_{x,N} + \sigma_{x,Mf} = 33,09 \text{ MPa}$$

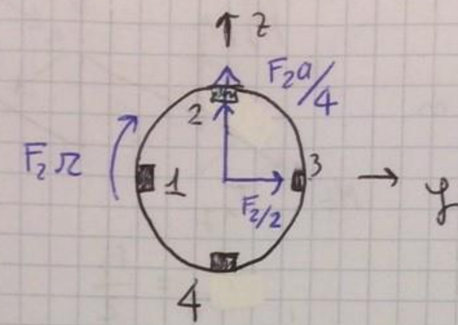
Punto 1:

$$\sigma_x = \sigma_{x,N} + \sigma_{x,Mf} = -1,26 \text{ MPa} - 34,31 \text{ MPa} = -35,54 \text{ MPa}$$

CASO 2 - (h - h)

$d_h = 15$ mm





Punto 1: $\tau_{Mt} = \dfrac{F_2 R \cdot d_h/2}{J_P} = \dfrac{(385 N \cdot 25\ mm) \cdot 15\ mm/2}{\dfrac{\pi d_h^4}{32}} = 14,52$ MPa

$\tau_T = 0$

$\sigma_x = \dfrac{M_y \cdot d_h/2}{J} = \dfrac{\dfrac{F_2 \cdot a}{4} \cdot \dfrac{d_h}{2}}{\dfrac{\pi d_h^4}{64}} = 20,33$ MPa

Punto 2 $\tau_{Mt} = 14,52$ MPa

$\tau_T = \dfrac{4}{3}\dfrac{T}{A} = 1,45$ MPa $\quad \tau_{Mt} + \tau_T = 15,97$ MPa

Punto 3 $\tau_{Mt} = 14,52$ MPa $\quad \tau_T = 0$

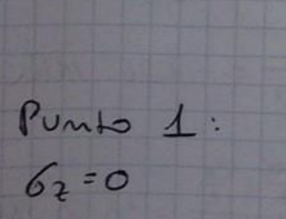
$\sigma_x = -20,33$ MPa

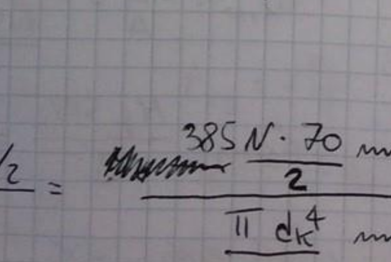
Punto 4 $\tau_{Mt} = 14,52$ MPa

$\tau_T = 1,45$ MPa $\quad \tau_{Mt} + \tau_T = -13,07$ MPa

CASO 2 (k - k)

$d_k = 20$ mm





Punto 1:

$\sigma_z = 0$

$\tau_{Mt(max)} = \dfrac{M_t \cdot d_k/2}{J_P} = \dfrac{\dfrac{385 N \cdot 70\ mm}{2} \cdot \dfrac{d_k}{2}}{\dfrac{\pi d_k^4}{32}\ mm^4} = 8,58$ MPa

$\tau_{T(max)} = \dfrac{4}{3}\dfrac{T}{A} = \dfrac{4}{3}\dfrac{F_2/2}{\pi d_k^2/4} = 0,82$ MPa

Punto 2:

$$\sigma_z = -\frac{M_y \cdot d_K/2}{J} = -\frac{\left(\frac{3}{4}F_2 \cdot R\right) d_K/2}{\frac{\pi d_K^4}{64}} = -9,2 \ MPa$$

$\tau_{max(M_t)} = 8,58 \ MPa$

Punto 3

$\tau_{M_t(max)} = 8,58 \ MPa$

$\tau_{(T)} = 0,82 \ MPa$

Punto 4:

$\sigma_z = 9,2 \ MPa$

$\tau_{M_t(max)} = 8,58 \ MPa$