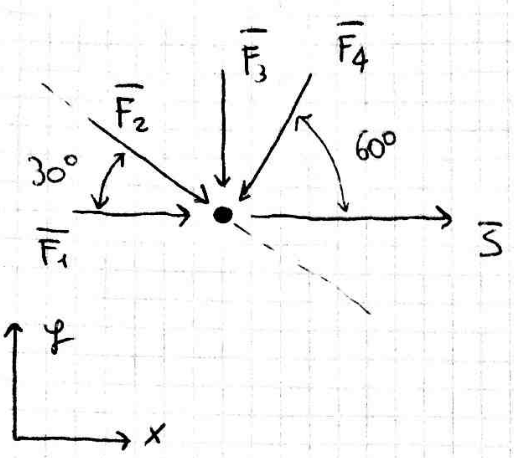
Esercizio 1



Il punto materiale P è soggetto a uno spostamento $\overline{S}$
Inoltre, sono applicate le seguenti forze:

$|\overline{F_1}| = 250\ N \rightsquigarrow F_1 = 250\ N$ Per comodità tolgo il segno di modulo

$F_2 = 500\ N$

$F_3 = 100\ N$

$F_4 = 300\ N$

$S = 25\ mm$

Lavoro ottenuto come sommatoria dei singoli contributi di ciascuna forza:

$$W_{tot} = \sum_{i=1}^{n} \overline{F_i} \cdot \overline{S} = W_{F_1} + W_{F_2} + W_{F_3} + W_{F_4}$$

$$W_{F_1} = \overline{F_1} \cdot \overline{S} = |\overline{F_1}||\overline{S}| \cdot \cos \widehat{F_1 S} = 250\ N \cdot 25\ mm \cdot 1 = 6{,}25\ Nm\ [J]$$

$$W_{F_2} = \overline{F_2} \cdot \overline{S} = |\overline{F_2}||\overline{S}| \cdot \cos \widehat{F_2 S} = 500\ N \cdot 25\ mm \cos 30° = 10{,}83\ J$$

$$W_{F_3} = \overline{F_3} \cdot \overline{S} = |\overline{F_3}||\overline{S}| \cdot \cos \widehat{F_3 S} = 0\ J$$

$$W_{F_4} = \overline{F_4} \cdot \overline{S} = |\overline{F_4}||\overline{S}| \cdot \cos \widehat{F_4 S} = 300\ N \cdot 25\ mm \cdot \cos\left(\pi - \frac{\pi}{3}\right) = -3{,}75\ J$$

$$W_{tot} = (6{,}25 + 10{,}83 + 0 - 3{,}75)\ J = 13{,}33\ J$$

Alternativa: calcoliamo la risultante agente sul punto $\overline{P}$:

$$\overline{F_1} = [250, 0, 0]$$

$$\overline{F_2} = \left[500\frac{\sqrt{3}}{2}, -500\frac{1}{2}, 0\right]$$

$$\overline{F_3} = [0, -100, 0]$$

$$\overline{F_4} = \left[-300\frac{1}{2}, -300\frac{\sqrt{3}}{2}, 0\right]$$

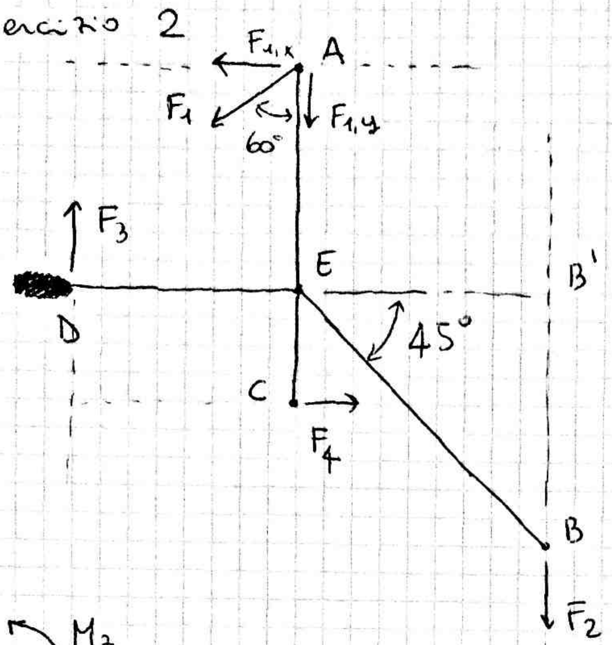
$$\overline{F_{tot}} = \overline{F_1} + \overline{F_2} + \overline{F_3} + \overline{F_4} = [533, -610, 0]$$

$$W_{tot} = \underbrace{\overline{F_{tot}} \cdot \overline{S}}$$

Si può calcolare in due modi:

- Sapendo che $\overline{S}$ ha componente solo lungo x:
  $W_{tot} = F_{tot,x} \cdot S = 533\ N \cdot 25\ mm = 13{,}33\ J$

Esercizio 2



$F_1 = 250\ N$

$F_2 = 500\ N$

$F_3 = 100\ N$

$F_4 = 300\ N$

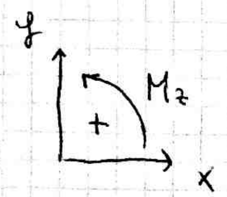
$AE = 500\ mm$

$BE = 750\ mm$

$CE = 250\ mm$

$DE = 500\ mm$

Calcolo della risultante delle forze e dei momenti in E e in C



$$\overline{R} = \sum_{i=1}^{m} \overline{F}_i \longrightarrow \begin{cases} R_x = \sum_{i=1}^{m} F_{x,i} \\ R_y = \sum_{i=1}^{m} F_{y,i} \end{cases}$$

$$\begin{cases} R_x = -F_1 \operatorname{sen}\frac{\pi}{3} + F_4 = -250\,N \cdot \frac{\sqrt{3}}{2} + 300\,N = 83,5\ N \\ R_y = -F_1 \cos\frac{\pi}{3} - F_2 + F_3 = -250\,N \cdot \frac{1}{2} - 500\,N + 100\,N = -525\,N \end{cases}$$

In termini di risultante di un vettore

$$\sum_{i=1}^{m} F_i(E) = \sum_{i=1}^{m} F_i(C)$$

$F_x = 83,5\ N$

$F_y = 525\ N$

Per i momenti utilizziamo il metodo 2 e il metodo 3

$$\sum_{i=1}^{m} M_{z,i}^{(C)} = M_{F_{1,z}}^{(C)} + M_{F_{2,z}}^{(C)} + M_{F_{3,z}}^{(C)} + M_{F_{4,z}}^{(C)} =$$

$$= + \overline{AC} \cdot F_{1x} - \overline{EB} \cos 45^\circ F_2 - \overline{DE}\, F_3 + 0 =$$

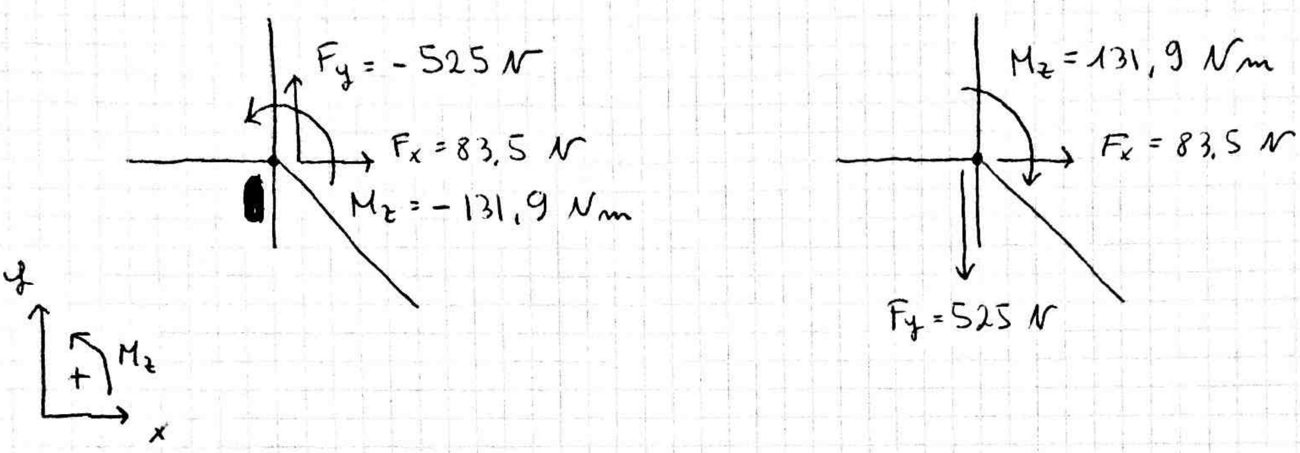
$$= 750\ mm \cdot 250 \operatorname{sen}\frac{\pi}{3} - 750 \cos 45^\circ \cdot 500\,N - 500\ mm\ 100\,N =$$

$$= -152,79\ Nm$$

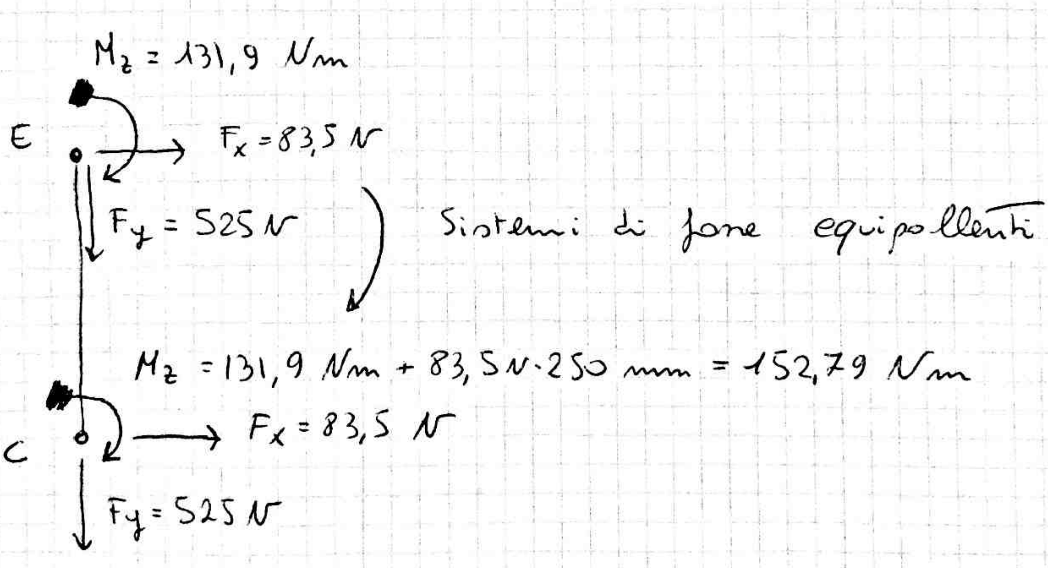
$$\sum_{i=1}^{n} M_{z,i}^{(E)} = \overline{AE}\, F_{1,x} - F_2\, \overline{EB} \cos 45^\circ - \overline{DE}\, F_3 + \overline{EC}\, F_4 =$$

$$= 500\ mm\ 250\ N \operatorname{sen}\frac{\pi}{3} - 750\ mm\ \frac{\sqrt{2}}{2}\ 500\ N - 500\ mm \cdot 100\ N +$$

$$+ 250\ mm \cdot 300\ N = -131,9\ Nm$$



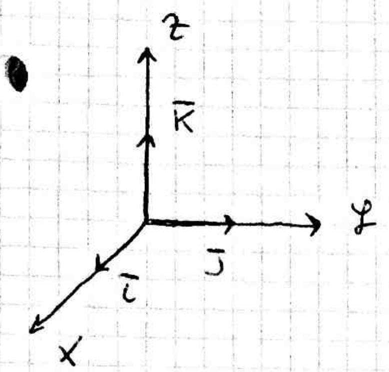
Metodo per trovare il sistema di forze equipollenti in C partendo dalla risultante in E

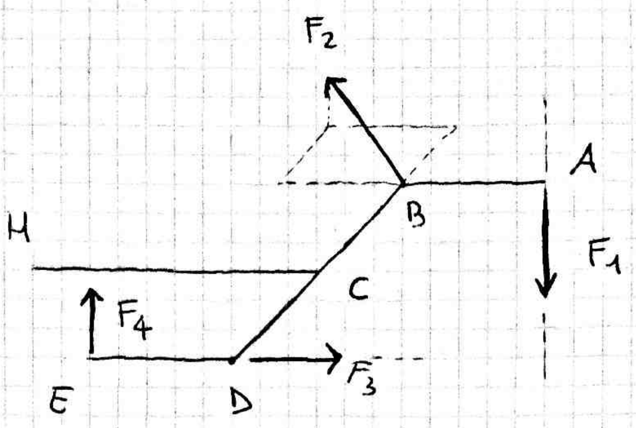


$R_C = R_E$

$M_C = M_E - \overline{EC}\, F_x = -131,9\ Nm - 250\ mm\ 83,5\ N = -152,8\ Nm$

Esercizio 3





$CH = 1000$ mm

$CD = 500$ mm

$BC = 500$ mm

$AB = 500$ mm

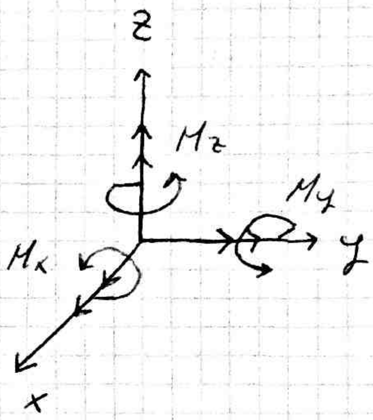
$ED = 500$ mm

$F_1 = \left[0, 0, -250 \right]$ N

$F_2 = \left[-100, -100, 100 \right]$ N

$F_3 = \left[0, 250, 0 \right]$ N

$F_4 = \left[0, 0, 250 \right]$ N



Calcolare la risultante delle forze e dei momenti in H

$$\bar{R} = \sum_{i=1}^{n} \bar{F}_i$$

$$\begin{cases} R_x = \sum_{i=1}^{n} F_{x,i} = (0 - 100 + 0 + 0)\,N = -100\,N \\ R_y = \sum_{i=1}^{n} F_{y,i} = F_{y,1} + F_{y,2} + F_{y,3} + F_{y,4} = (0 - 100 + 250 + 0)\,N = 150\,N \\ R_z = \sum_{i=1}^{n} F_{z,i} = (-250 + 100 + 0 + 250)\,N = 100\,N \end{cases}$$

$$\bar{M}_{tot}^{(H)} = \bar{M}_{F_1}^{(H)} + \bar{M}_{F_2}^{(H)} + \bar{M}_{F_3}^{(H)} + \bar{M}_{F_4}^{(H)}$$

Metodo 1

$$\bar{M} = \bar{r} \times \bar{F}$$

$$\bar{M}_{F_1}^{(H)} = \overline{A-H} \times \bar{F}_1$$

$$\overline{A-H} = \overline{C-H} + \overline{B-C} + \overline{A-B} = \begin{bmatrix} 0 \\ 1000 \\ 0 \end{bmatrix} + \begin{bmatrix} -500 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 500 \\ 0 \end{bmatrix} = \begin{bmatrix} -500 \\ 1500 \\ 0 \end{bmatrix}$$

$$\overline{M}_{F_1}^{(H)} = \overline{A-H} \times \overline{F_1} = \det \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -500 & 1500 & 0 \\ 0 & 0 & -250 \end{vmatrix} \begin{matrix} \overline{i} & \overline{j} \\ -500 & 1500 \\ 0 & 0 \end{matrix} =$$

$$= \left(-250\,N \cdot 1500\,mm\,\overline{i}\right) - \left(250\,N \cdot 500\,mm\,\overline{j}\right) =$$

$$= -375\,Nm\,\overline{i} - 125\,Nm\,\overline{j}$$

$$\overline{M}_{F_2}^{(H)} = \overline{B-H} \times \overline{F_2}$$

$$\overline{B-H} = \overline{C-H} + \overline{B-C} = \begin{bmatrix} 0 \\ 1000 \\ 0 \end{bmatrix} + \begin{bmatrix} -500 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -500 \\ 1000 \\ 0 \end{bmatrix}$$

$$\overline{M}_{F_2}^{(H)} = \det \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ -500 & 1000 & 0 \\ -100 & -100 & 100 \end{vmatrix} \begin{vmatrix} \overline{i} & \overline{j} \\ -500 & 1000 \\ -100 & -100 \end{vmatrix} =$$

$$= \left(100\,N\,1\,m\,\overline{i} + 100\,N\,0{,}5\,m\,\overline{k}\right) - \left(-100\,N\,1\,m\,\overline{k} - 100\,N\,0{,}5\,m\,\overline{j}\right) =$$

$$= 100\,Nm\,\overline{i} + 150\,Nm\,\overline{k} + 50\,Nm\,\overline{j}$$

$$\overline{M}_{F_3}^{(H)} = \overline{D-H} \times \overline{F_3}$$

$$\overline{D-H} = \overline{C-H} + \overline{D-C} = \begin{bmatrix} 0 \\ 1000 \\ 0 \end{bmatrix} + \begin{bmatrix} 500 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 500 \\ 1000 \\ 0 \end{bmatrix}$$

$$\overline{M}_{F_3}^{(H)} = \det \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 500 & 1000 & 0 \\ 0 & 250 & 0 \end{vmatrix} \begin{matrix} \overline{i} & \overline{j} \\ 500 & 1000 \\ 0 & 250 \end{matrix} =$$

$$= \left(250\,N \cdot 0{,}5\,m\,\overline{k}\right) - \left(0\right) = 125\,Nm\,\overline{k}$$

$$\overline{M}_{F_4}^{(H)} = \overline{E-H} \times \overline{F_4}$$

$$\overline{E-H} = \overline{C-H} + \overline{D-C} + \overline{E-D} = \begin{bmatrix} 0 \\ 1000 \\ 0 \end{bmatrix} + \begin{bmatrix} 500 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -500 \\ 0 \end{bmatrix} = \begin{bmatrix} 500 \\ 500 \\ 0 \end{bmatrix}$$

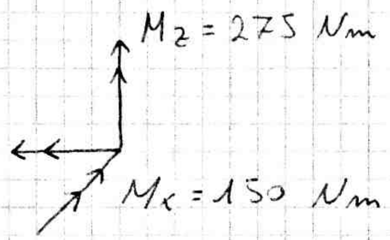
$$\overline{M}_{F_4}^{(H)} = \det \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 500 & 500 & 0 \\ 0 & 0 & 250 \end{vmatrix} \begin{vmatrix} \bar{i} & \bar{j} \\ 500 & 500 \\ 0 & 0 \end{vmatrix} =$$

$$= \left(250\,N \cdot 0{,}5\,m\ \bar{i}\right) - \left(250\,N \cdot 0{,}5\,m\ \bar{j}\right) =$$

$$= 125\,Nm\ \bar{i} - 125\,Nm\ \bar{j}$$

$$\overline{M}_{tot}^{(H)} = \begin{bmatrix} (-375 + 100 + 125)\,Nm\ \bar{i} \\ (-125 + 50 - 125)\,Nm\ \bar{j} \\ (150 + 125)\,Nm\ \bar{k} \end{bmatrix} = \left(-150\,\bar{i} - 200\,\bar{j} + 275\,\bar{k}\right) Nm$$

Punto H



Metodo 2 (e 3)

$$\overline{M}_{x}^{(H)} = -\left(\overline{CH} + \overline{AB}\right) F_1 + \overline{CH} \cdot F_{2,z} + \left(\overline{CH} - \overline{ED}\right) F_4 =$$

$$= -1{,}5\,m\ 250\,N + 1\,m \cdot 100\,N + 0{,}5\,m\ 250\,N = -150\,Nm\ \bar{i}$$

$$\overline{M}_{y}^{(H)} = -\overline{BC} \cdot F_1 + \overline{BC} \cdot F_{2,z} - \overline{DC}\,F_4 =$$

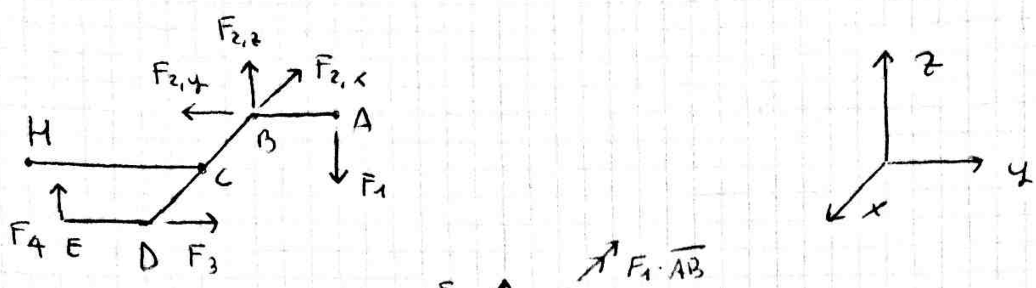
$$= -0{,}5\,m\ 250\,N + 0{,}5\,m\ 100\,N - 0{,}5\,m\ 250\,N = -200\,Nm\ \bar{j}$$

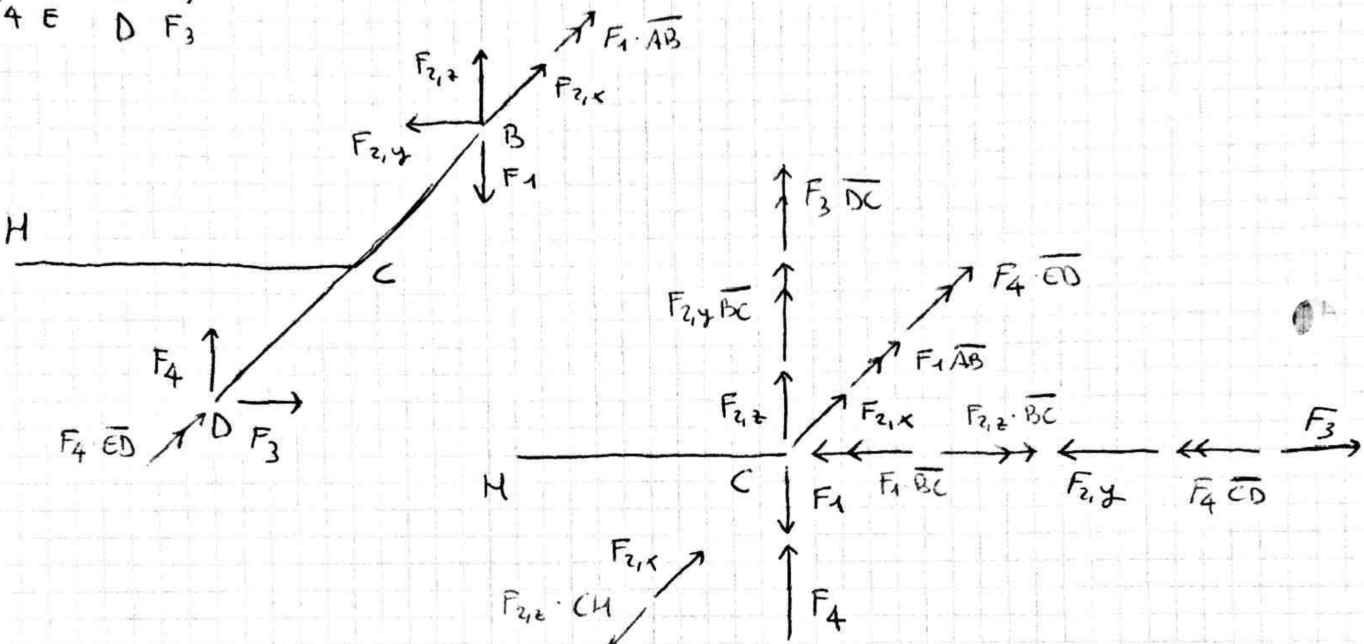
$$\overline{M}_{z}^{(H)} = +\overline{CH}\ F_{2,x} + \overline{BC}\ F_{2,y} + \overline{DC}\ F_3 =$$

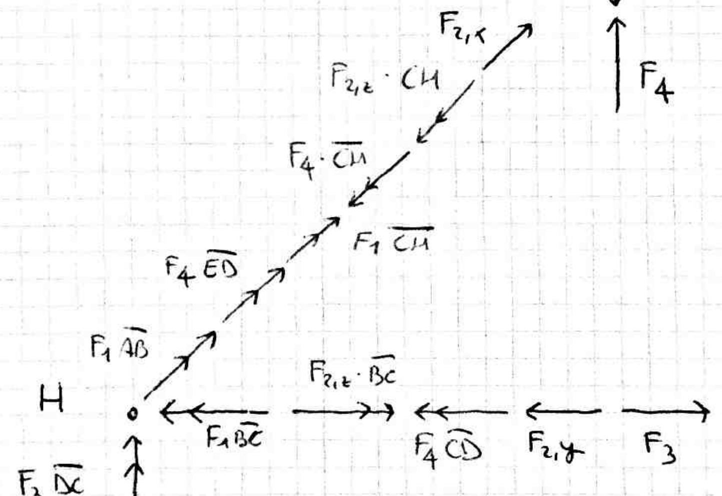
$$= 1\,m\ 100\,N + 0{,}5\,m\ 100\,N + 0{,}5\,m\ 250\,N = 275\,Nm\ \bar{k}$$

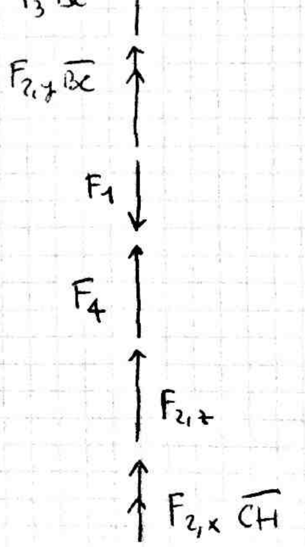
Metodo 4

→ Trasporto dei momenti e delle forze









Questi sistemi di forze sono equipollenti tra di loro, poiché presi un punto generico della struttura presentano gli stessi (o le stesse) risultanti in termini di $\vec{F}$ e di $\vec{M}$