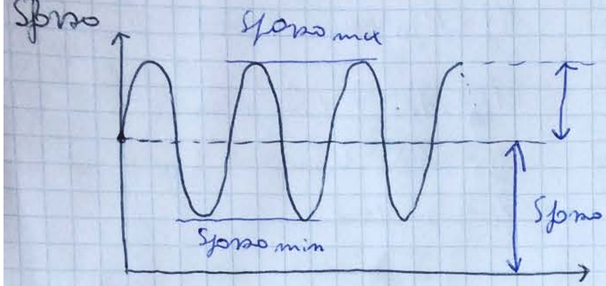
ANALISI E VERIFICA A FATICA

Monoassiale

1 - Analisi dello stato di sforzo e della variazione nel tempo delle componenti



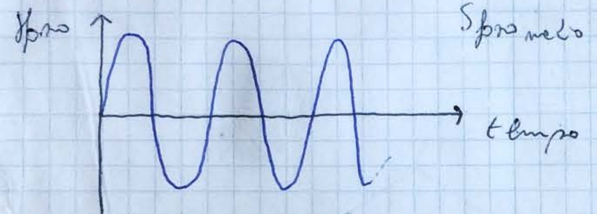
$$\text{Sforzo}_{,a} = \frac{\text{Sforzo}_{,max} - \text{Sforzo}_{,min}}{2}$$

$$\text{Sforzo medio} = \frac{\text{Sforzo}_{max} + \text{Sforzo}_{min}}{2}$$

Si definisce rapporto di ciclo

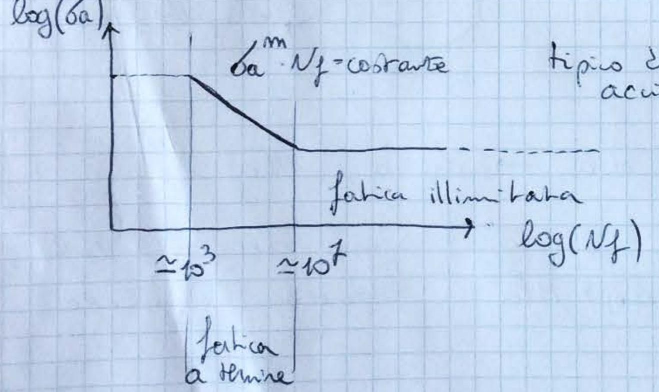
$$R = \frac{\text{Sforzo min}}{\text{Sforzo max}}$$

Fatica alternata: $R = -1$



Fatica pulsante: $R = 0$

2 - Stima del limite di fatica



tipico diagramma di ~~[illegible]~~ Wöhler per acciai

→ Definito per un singolo rapporto di ciclo R

Definiamo: $R = -1$

σ_{FAf}: limite di fatica alternato a flessione

σ_{FAa}: limite di fatica alternata assiale

τ_{FAt}: limite di fatica alternata a torsione

Acciai duttili:

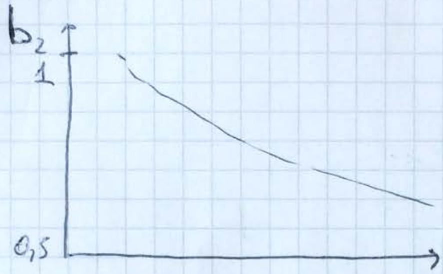
$\sigma_{FAf} \cong (0{,}4 \div 0{,}6) R_m \cong 0{,}5 R_m$; $\sigma_{FAa} \cong (0{,}3 \div 0{,}45) R_m \cong 0{,}4 \cdot R_m$; $\tau_{FAt} = (0{,}23 \div 0{,}33) R_m = 0{,}25 \cdot R_m$

limiti di fatica per

$R = -1$; $K_t = 1$; $R_t = 0{,}3\ \mu m$ → Condizioni provino di prova

3 ~~4~~ - Effetto dimensionale : Provino → Componente reale

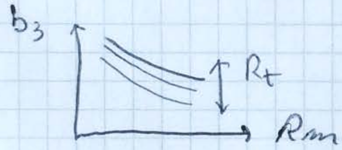
b_2 :



$b_2 < 1$

4 ~~5~~ - Effetto rugosità : Provino ($R_t = 0{,}3\ \mu m$) → Componente

b_3 :



$b_3 < 1$

5 ~~6~~ - Effetto presenza intagli K_f in generale $K_f < K_t$

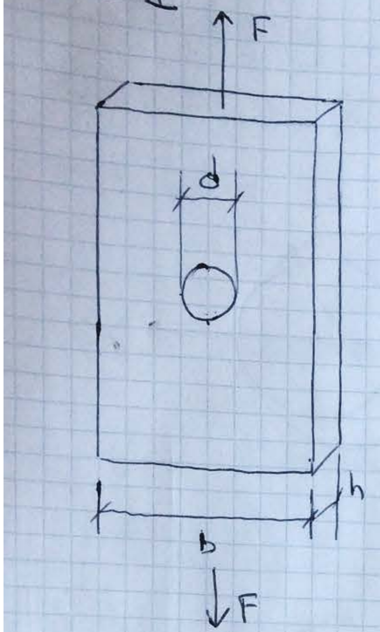
$K_f = 1 + q(K_t - 1)$ q: sensibilità all'intaglio $q = 1$ massima, $q = 0$ nulla

Neuber: $q = \dfrac{1}{1 + \sqrt{e/r}}$

7 - Verifica a fatica : $\sigma_a \le \sigma_{a,amm} = \dfrac{\sigma_{a,lim}}{\eta} = \dfrac{\sigma_{FA} \cdot b_2 \cdot b_3}{\eta\, K_f}$

Il punto 6 considera l’effetto dello sforzo medio (tramite diagramma di Haigh) che è trattato nella seconda parte della lezione.

Esercizio 1



$d = 30$ mm
$b = 120$ mm
$h = 20$ mm

- Carico assiale con andamento sinusoidale:

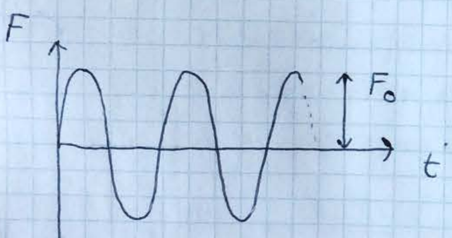
$$F = F_0 \sin(\omega t)$$

- No problemi di instabilità
- Max F_0 che consente durata illimitata sulla base dei seguenti materiali:

Materiale	R_m (MPa)	σ_{FAa}
C20 bon	500	200
C45 bon	750	300
39NiCrMo4 bon	1000	400

- Si prevedono tre diverse finiture superficiali del foro:

$R_t = 3{,}2 - 6{,}3 - 10\ \mu m$



$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$R = -1$

limite di fatica alternata assiale:

$$\sigma_{FAa}' = \frac{\sigma_{FAa} \cdot b_2 \cdot b_3}{K_{fa}}$$

$b_2 = 1$ in caso di fatica assiale l'effetto dimensionale è trascurabile

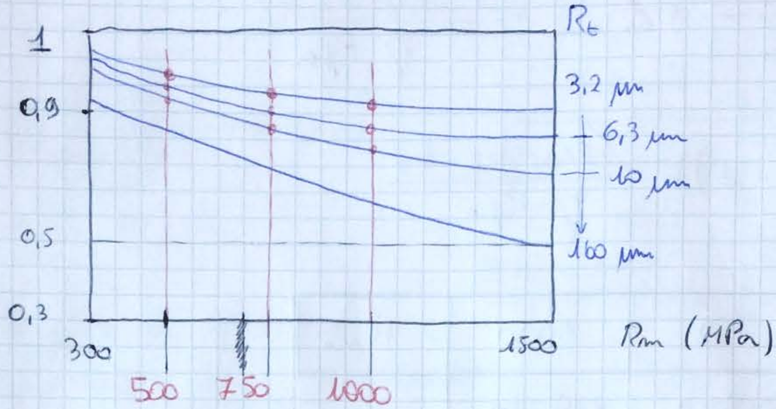
b_3: coefficiente che tiene conto dell'effetto della finitura della superficie

$K_{fa} = 1 + q\,(K_{t,a} - 1)$ con $q = \dfrac{1}{1 + \sqrt{\rho/r}}$ (sensibilità all'intaglio secondo Neuber)

$K_{ta} = 2{,}45$ (si veda tabella)

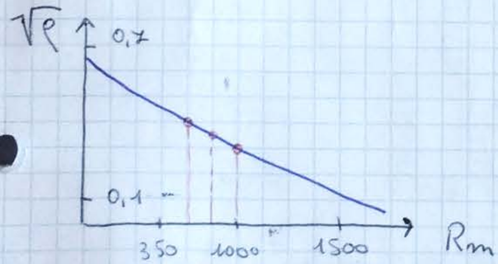
$(d/b = 0{,}25)$

b_3 : si deve guardare il grafico (funzione anche di R_m)



b_3	500	750	1000	R_m (MPa)
3,2	0,95	0,93	0,92	
6,3	0,93	0,89	0,88	
10	0,89	0,84	0,8	
R_t (μm)				

K_{fa} : Coefficiente di intaglio a fatica



	500	750	1000	R_m
$\sqrt{\rho}$	0,4	0,29	0,2	
q	0,906	0,93	0,95	
K_{fa}	2,314	2,349	2,379	
σ_{FAa}	200	300	400	

Materiale : acciai duttili

$$\sigma_{FAa} \cong (0{,}3 \div 0{,}45) R_m \cong 0{,}4 \cdot R_m$$

si assume : $\eta \cong 2$

$F = F_0 \operatorname{sen}(\omega t)$

$$\sigma_{nom} = \frac{F}{(b-d)h} = \frac{F_0 \operatorname{sen}(\omega t)}{(b-d)h}$$

$$\sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta} = \frac{\sigma_{FAa} \cdot b_2 \cdot b_3}{\eta \, K_{fa}}$$

$$\sigma_a = \frac{F_0}{(b-d)h} \leq \sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta}$$

$R = -1$

Condizione limite

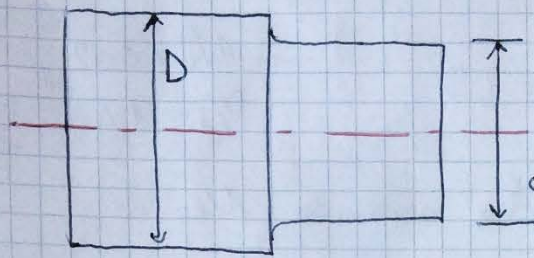
$$\sigma_{a,max} = \frac{F_{0,max}}{(b-d)h} = \sigma_{a,amm} = \frac{\sigma_{FAa} \cdot b_2 \cdot b_3}{\eta K_{fa}}$$

$$\Rightarrow \quad F_{0,max} = (b-d)h \frac{\sigma_{FAa} \cdot b_2 \cdot b_3}{\eta K_{fa}}$$

$F_{0,max}$	500	750	1000	R_m
3,2	73898	106900	139220	
6,3	72342	102300	133170	
10	69231	96552	121060	
R_t				

σ_{FA}	500	750	1000	R_m
3,2	82,11	118,8	154,7	
6,3	80,4	113,7	148	
10	76,9	107,3	134,5	
R_t				

Esercizio 2



$D = 70$ mm

$d = 50$ mm

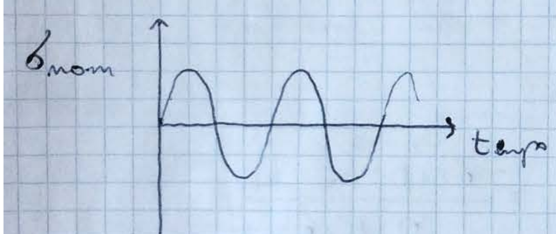
$r = 1$ mm

Materiale	R_m (MPa)	σ_{AFf}
C10 temp. acqua	450	225
C30 bon	550	275
30NiCrMo5 temprato + disteso	1750	875

$R_t = (3,2 - 6,3 - 10)\ \mu m$

Albero soggetto a flessione rotante presenta una variazione di sezione dovuta all'alloggiamento di un cuscinetto

↳ Calcolare il massimo valore di M_f che consenta durata illimitata.



$$\sigma_{nom} = \frac{M_f \cdot d/2}{\frac{\pi d^4}{64}} = \sigma_{a,nom} \cdot \text{sen}(\omega t)$$

$$\sigma'_{FAf} = \frac{\sigma_{FAf} \cdot b_2 \cdot b_3}{K_{ff}}$$

Acciaio duttile: $\sigma_{FAf} \cong (0,4 \div 0,6) R_m \cong 0,5 \cdot R_m$

b_2: Effetto dimensionale $\cong 0,82$ (per il caso della flessione alternata) (tabella)

b_3: Effetto finitura superficiale (vedere tabella)

Materiale	R_m	$R_t = 3,2\ \mu m$	$6,3\ \mu m$	$10\ \mu m$
C10	450	0,96	0,94	0,91
C30	550	0,95	0,92	0,88
30 NiCrMo5	1750	0,92	0,87	0,75

b_3

K_{ff} : Coefficiente di intaglio a fatica a flessione

$$K_{ff} = 1 + q\left(K_{tf} - 1\right) \qquad K_{tf} = 3,5 \qquad r/d = 0,02 \quad D/d = 1,4$$

$$q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}}$$ sensibilità all' intaglio secondo Neuber

R_m	$\sqrt{\rho}$	q	K_{ff}
450	0,47	0,68	2,70
550	0,42	0,70	2,76
1750	0,05	0,95	3,38

$$\sigma_{nom} = \frac{M_f \cdot d/2}{J} = \sigma_{a,nom} \operatorname{sen}(\omega t)$$

$$M_f = \frac{J}{d/2}\,\sigma_a \operatorname{sen}(\omega t) = M_{f,a} \operatorname{sen}(\omega t)$$

$$\sigma_{a,nom} = \frac{M_{f,a}\, d/2}{J} \leq \sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta} = \frac{\sigma_{FAf} \cdot b_2 \cdot b_3}{\eta\, K_{ff}}$$

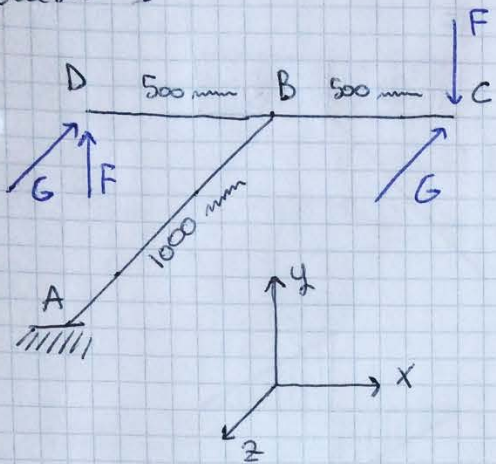
Condizione limite: $\sigma_{a,max} = \dfrac{M_{f,a,max}\, d/2}{J} = \dfrac{\sigma_{FAf} \cdot b_2 \cdot b_3}{\eta \cdot K_{ff}}$

$$\Rightarrow M_{f\,a,max} = \frac{J \cdot \sigma_{FAf} \cdot b_2 \cdot b_3}{d/2\ \eta \cdot K_{ff}} \qquad \eta = 2$$

(Nm)

$M_{f\,a,max}$	450	550	1750	R_m
3,2	402	476	1198	
6,3	394	461	1133	
10	381	441	977	
R_t				

Esercizio 3



- Sezione della trave circolare con diametro $d = 40$ mm
- Nel punto A c'è una variazione di sezione che produce un K_t coefficiente di sovrasollecitazione – ($K_t = 1,5$)

↳ Azioni interne, separate per F e G

↳ Punto più sollecitato delle due aste ed effettuare verifica di resistenza statica

$F = 1000$ N ; $G = 3000$ N

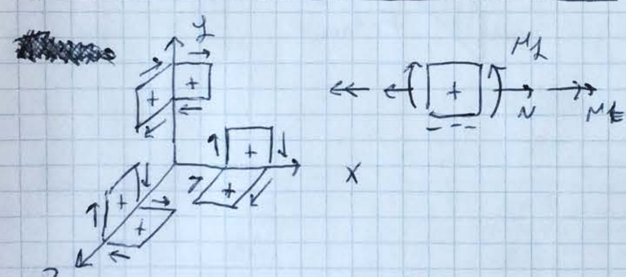
↳ 30NiCrMo3, $R_m = 900$ MPa ; $R_s = 600$ MPa

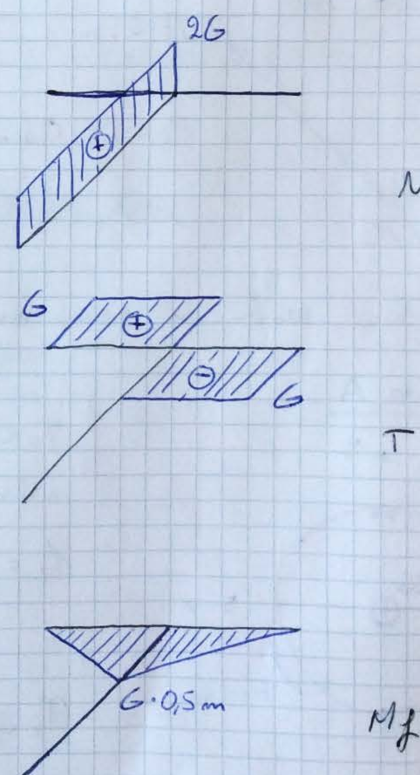
↳ Effettuare verifica a fatica della sezione A considerando:

$G = 0$

$F = F_0 \operatorname{sen}(\omega t)$ con $F_0 = 1000$ N

Azioni Interne dovute a G



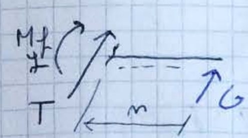


Tramo $\overline{DB}$

$T = G$

$M_{fy} = Gm$

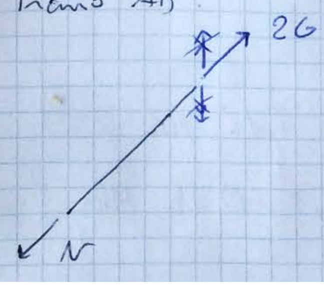
Tramo $\overline{BC}$



$T = -G$

$M_{fy} = Gm$

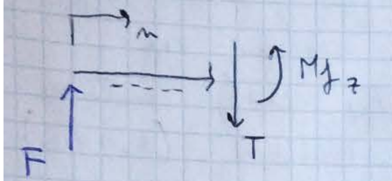
Tramo $\overline{AB}$:



$N = 2G$

Azioni Interne dovute a F

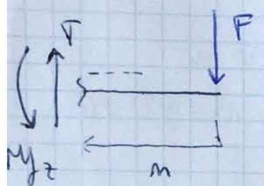
Tratto $\overline{DB}$:



$T = F$

$Mf_z = F\,m$

Tratto $\overline{BC}$

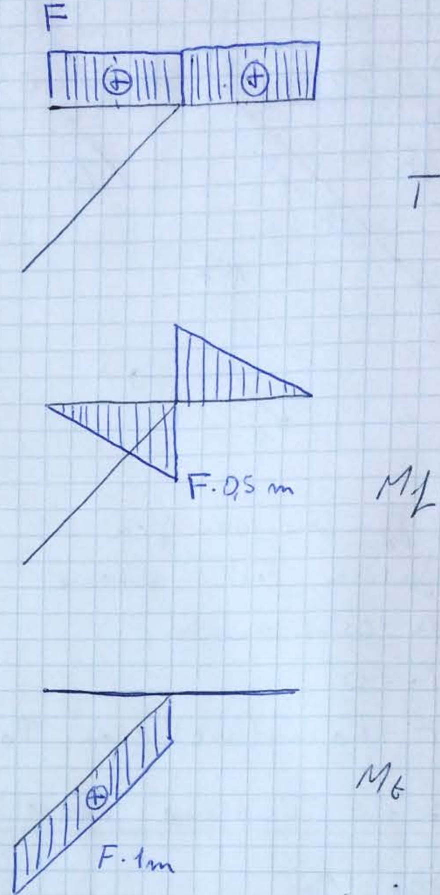


$T = F$

$Mf_z = F\,m$

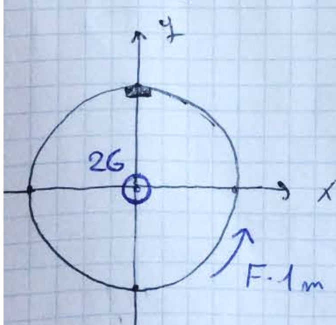
Tratto $\overline{AB}$

$M_t = 2F \cdot 0{,}5\ m$



I due punti da verificare sono il punto A e il punto B

Punto A



Tutti i punti sul diametro esterno sono nelle stesse condizioni

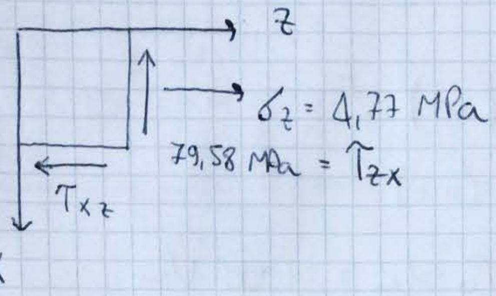
$$\sigma_z = \frac{2G}{A} = \frac{2G}{\frac{\pi d^2}{4}} = \frac{2 \cdot 3000\,N}{\frac{\pi\, 40^2\,mm^2}{4}} = 4{,}77\ MPa$$

$$\tau_{zx} = \frac{M_t \cdot d/2}{\frac{\pi d^4}{32}} = \frac{F \cdot 1m \cdot 40\,mm/2}{\frac{\pi\, 40^4\,mm^4}{32}} = 79{,}58\ MPa$$

$$\bar{\bar{\sigma}} = \begin{bmatrix} 0 & 0 & -79{,}58 \\ 0 & 0 & 0 \\ -79{,}58 & 0 & 4{,}77\ MPa \end{bmatrix}$$

$\sigma_{z,max} = K_t \cdot \sigma_z = 7{,}16\ MPa$

$\tau_{zx,max} = K_t \cdot \tau_{zx} = 119{,}37\ MPa$



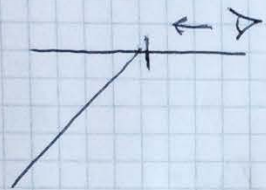
Materiale duttile

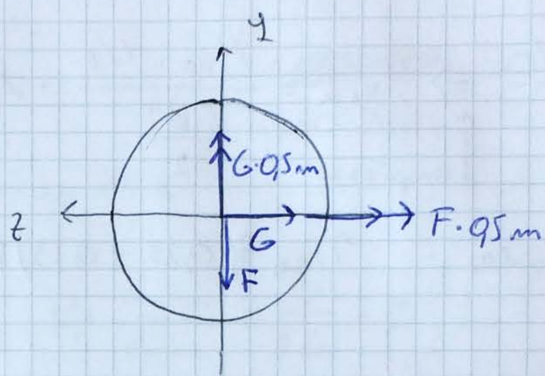
→ uso ad esempio Von Mises:

$$\sigma_{VM} = \sqrt{\sigma^2 + 3\tau^2} = 206{,}9 \text{ MPa}$$

$$\eta_{VM} = \frac{\sigma_{lim}}{\sigma_{VM}} = \frac{R_s}{\sigma_{VM}} = \frac{600 \text{ MPa}}{206{,}9 \text{ MPa}} = 2{,}9$$

Punto B





→ Trascuro il taglio

$$\vec{M_{f\,tot}} = \vec{M_{f\,G}} + \vec{M_{f\,F}} \qquad |M_{f\,tot}| = \sqrt{(G \cdot 0{,}5\,m)^2 + (F \cdot 0{,}5\,m)^2} =$$

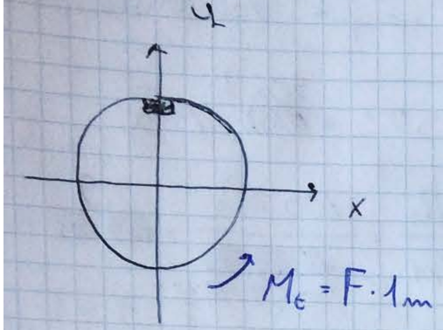
$$= 1{,}5811 \cdot 10^6 \text{ Nmm}$$

$$\sigma_{x,max} = \frac{M_f \cdot d/2}{\frac{\pi d^4}{64}} = \frac{1{,}5811 \cdot 40/2 \text{ mm}}{\frac{\pi\, 40^4 \text{ mm}^4}{64}} = 251{,}65 \text{ MPa}$$

$$\sigma_{VM} = \sigma_{x,max} = 251{,}65 \text{ MPa} \leq \sigma_{amm} = \frac{\sigma_{lim}}{\eta_{VM}}$$

$$\eta_{VM} = \frac{\sigma_{lim}}{\sigma_{VM}} = \frac{600 \text{ MPa}}{251{,}65 \text{ MPa}} = 2{,}38$$

Verifica a fatica nel punto A



$$F = F_0 \, sen(\omega t)$$

$$M_t = M_{t0} \, sen(\omega t)$$

con $M_{t0} = F_0 \cdot 1m = 1000 \, Nm$

$$\tau_{zx} = \frac{M_t \cdot d/2}{\frac{\pi d^4}{32}} \Rightarrow \tau_{zx,a} = \frac{M_{t0} \cdot d/2}{\frac{\pi d^4}{32}} = 79{,}6 \text{ MPa}$$

↑ ampiezza

Acciaio duttile: $\tau_{FAt} \cong (0{,}23 \div 0{,}33) R_m \cong 0{,}25 \cdot R_m =$
$= 225 \text{ MPa}$

Effetto dimensionale:

$b_2 = 0{,}85$ (tabella)

Finitura superficiale: (Ipotesi: Rettificato fine $R_t \simeq 3{,}2 \, \mu m$)

$b_3 = 0{,}92$

$K_t = 1{,}5$ (piuttosto basso, in generale)

$$K_{ft} = 1 + q(K_t - 1) \qquad q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}} \qquad \sqrt{\rho} = 0{,}22$$

$q = 0{,}89$

$r/d \simeq 0{,}02 \rightarrow r \cong 3 \, mm$
(tabella)

$$K_{ft} = 1 + 0{,}89(1{,}5 - 1) = 1{,}45$$

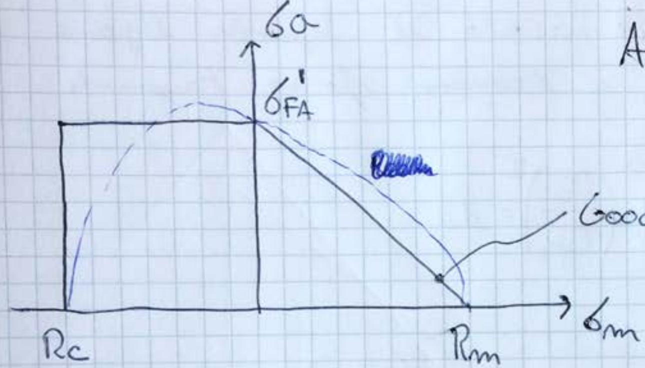
$$\tau'_{FAt} = \frac{\tau_{FAt} \cdot b_2 \cdot b_3}{K_{ft}} = \frac{225 \text{ MPa} \cdot 0{,}85 \cdot 0{,}92}{1{,}45} = 121 \text{ MPa}$$

$$\tau_{zx,a} \le \tau_{a,amm} = \frac{\tau_{a,lim}}{\eta}$$

Condizione limite: $\eta = \dfrac{\tau_{a,lim}}{\tau_{zx,a}} = \dfrac{\tau'_{FAt}}{\tau_{zx,a}} = 1{,}52 < 2$

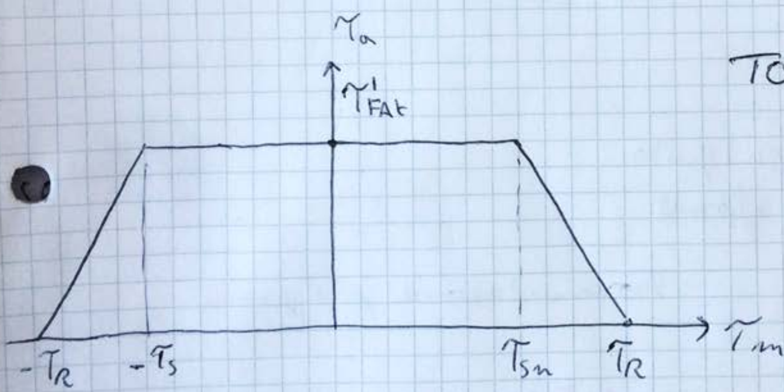
6- Correzione in caso $R \neq -1$: Diagramma di Haigh

ASSIALE / FLESSIONE



Goodman $\frac{\sigma_a}{\sigma'_{FA}} + \frac{\sigma_m}{R_m} = 1$

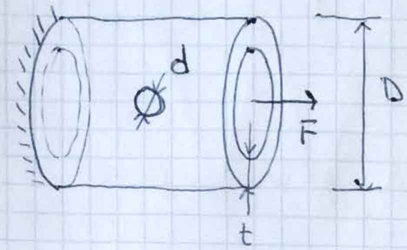
TORSIONE

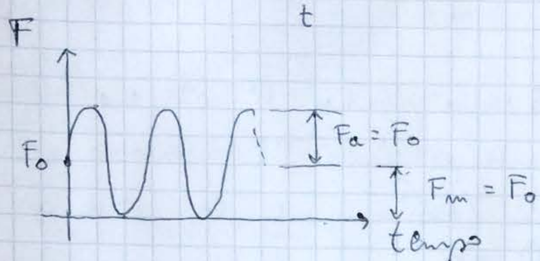


$\tau_R = 0,77\, R_m$

$\tau_{SM} = \frac{R_s}{\sqrt{3}}$

Esercizio 4





$D = 40$ mm ; $d = 40 - 2t = 32$ mm

$t = 4$ mm

$F = F_0 \, sen(\omega t) + F_0$

$d_f = 6$ mm

$F_0 = 25$ kN

Materiale: C10 temprato in acqua

$R_m = 500$ MPa

$R_s = 280$ MPa

$$\bar{\sigma}_a = \frac{F_0}{A} = \frac{F_0}{\frac{\pi(D^2 - d^2)}{4}} = 55{,}26 \text{ MPa}$$

$$\bar{\sigma}_m = 55{,}26 \text{ MPa} \qquad R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{0}{2\sigma_a} = \emptyset$$

Calcoliamo σ_{FAa} (determinato per $R = -1$)

Materiale duttile: $\sigma_{FAa} = 0{,}4 \cdot R_m = 0{,}4 \cdot 500 \text{ MPa} = 200$ MPa

Verifica a fatica:

$$\sigma_a \le \frac{\sigma_{a,lim}}{\eta}$$

$$\sigma'_{FAa} = \frac{\sigma_{FAa} \cdot b_2 \cdot b_3}{K_f} = \frac{200 \text{ MPa} \cdot 1 \cdot 0{,}85}{2{,}5} =$$

$$\sigma'_{FAa} = 68 \text{ MPa}$$

b_2: effetto dimensionale (caso assiale)

$b_2 = 1$

b_3: effetto finitura superficiale

$b_3 = 0{,}85$ (non viene specificato, quindi stiamo conservativi)

K_t: tabella per piastre $\quad H = \pi \cdot D = 126$ mm

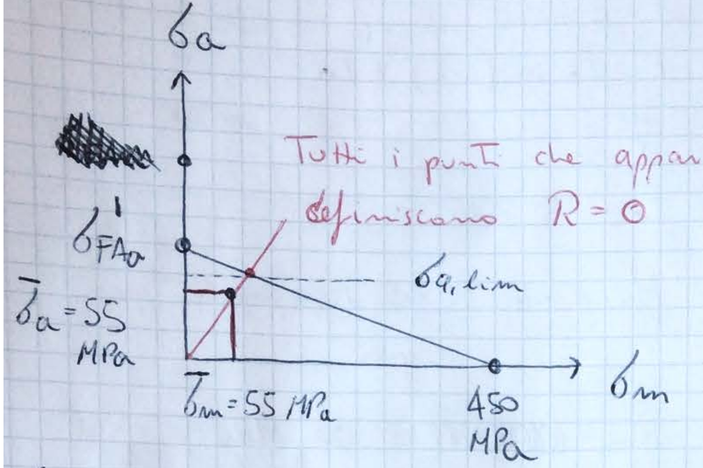
$d/H = 0{,}05$

$K_t = 2{,}85$

$$q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}} = \frac{1}{1 + \frac{0{,}4}{\sqrt{3\,mm}}} = 0{,}81$$

$$K_{fa} = 1 + q(K_t - 1) = 2{,}5$$

Diagramma di Haigh (effetto dello sforzo medio)



Questo punto si ottiene interpolando l'equazione di Goodman e la retta a $R = 0$

$$\begin{cases} \dfrac{\sigma_a}{\sigma'_{FAa}} + \dfrac{\sigma_m}{R_m} = 1 & (\text{Goodman}) \\ \sigma_a = \dfrac{\overline{\sigma}_a}{\overline{\sigma}_m}\,\sigma_m & (R=0) \end{cases} \quad \rightarrow \quad \sigma_m = \sigma_a \frac{\overline{\sigma}_m}{\overline{\sigma}_a} = \sigma_{a,lim}$$

nel punto di intersezione, in aggiunta $\overline{\sigma}_m = \overline{\sigma}_a$

[redacted]

$$\frac{\sigma_{a,lim}}{\sigma'_{FAa}} + \frac{\sigma_{a,lim}}{R_m} = 1 \qquad \sigma_{a,lim} = \frac{1}{\left(\dfrac{1}{\sigma'_{FAa}} + \dfrac{1}{R_m}\right)} =$$

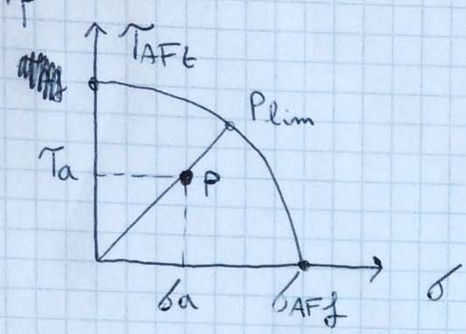
$$= \frac{1}{\dfrac{1}{68\,\text{MPa}} + \dfrac{1}{450\,\text{MPa}}} = 59 \text{ MPa}$$

$$\overline{\sigma}_a \le \sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta} \rightarrow \quad \eta = \frac{\sigma_{a,lim}}{\sigma_a} = \frac{59\text{ MPa}}{55\text{ MPa}} = 1{,}07$$

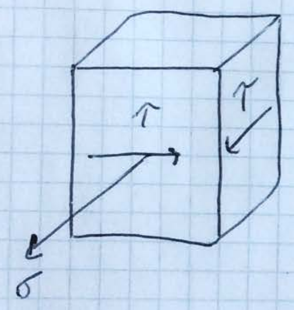
Non verificato!

CRITERIO DI FATICA MULTIASSIALE: GOUGH-POLLARD

Dagli esperimenti si è notato il seguente legame tra ampiezza di sforzo σ_a e sforzo tangenziale τ_a:



$$\left(\frac{\sigma_a}{\sigma_{AFf}}\right)^2+\left(\frac{\tau_a}{\tau_{AFt}}\right)^2=1$$



Vale solo per questo stato di sforzo

Formulazione più generica

Se σ e τ sono entrambe variabili nel tempo

$$\sigma_{GP}^* = \sqrt{\sigma_a^2+H^2\tau_a^2} \le \sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta}$$

con $H=\dfrac{\sigma_{a,lim}}{\tau_{a,lim}}$

Se $\tau = \text{costante} = \tau_m$

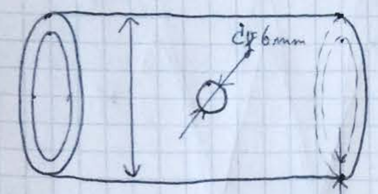
$$\sigma_{GP}^* = \sqrt{\sigma_a^2+H^2\tau_m^2} \le \sigma_{a,amm} = \frac{\sigma_{a,lim}}{\eta}$$

$$H=\frac{\sigma_{a,lim}}{\tau_{lim}} \qquad \tau_{lim}=\frac{R_s}{\sqrt{3}}$$

Specifica nel caso di τ costante

Esercizio 5

$d = 32\ mm$

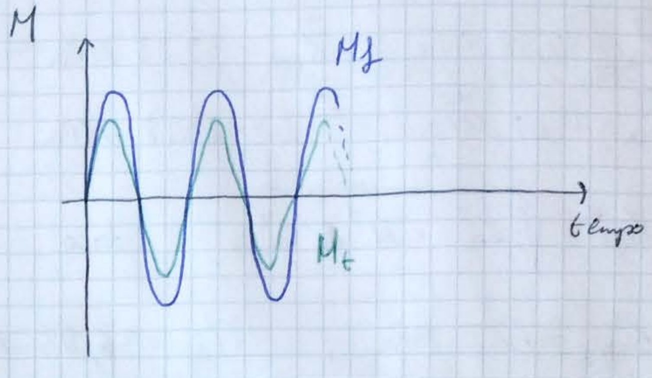


$D = 40\ mm$ $t = 4\ mm$

$M_f = M_{fo} \sin(\omega t)$

$M_t = M_{to} \sin(\omega t)$

η_{fatica} ?



Dati: $M_{fo} = 120\ Nm$

$M_{to} = 100\ Nm$

$R_m = 450\ MPa$

$R_s = 280\ MPa$

[redacted] $K_{ff} = 2,4$

$K_{ft} = 2,2$

$M_{f,a} = M_{fo} = 120\ Nm$

$M_{t,a} = M_{to} = 100\ Nm$

$$\Rightarrow \sigma_a = \frac{M_{f,a} \cdot d/2}{\frac{\pi (D^4 - d^4)}{64}} = \frac{M_{fo} \cdot d/2}{\frac{\pi (D^4 - d^4)}{64}} = 32,35\ MPa$$

$$\tau_a = \frac{M_{t,a} \cdot d/2}{\frac{\pi (D^4 - d^4)}{32}} = 13,48\ MPa$$

$\sigma_m = \tau_m = 0$

$$\sigma_{GP}^* = \sqrt{\sigma_a^2 + H^2 \tau_a^2} \leq \frac{\sigma_{a,lim}}{\eta} \quad \text{dove} \quad H = \frac{\sigma_{a,lim}}{\tau_{a,lim}}$$

$$\sigma_{a,lim} = \sigma'_{FAf} = \frac{\sigma_{FAf} \cdot b_2 \cdot b_3}{K_{ff}}$$

$\sigma_{FAf} \cong 0,5 \cdot R_m = 225\ MPa$

$b_2 = 0,85$ (tabella)

$b_3 = 0,85$

$K_{ff} = 2,4$

$$\sigma'_{FAf} = \frac{225 \cdot 0,85 \cdot 0,85}{2,4} = 67,7\ MPa$$

$$\tau_{a,lim} = \tau'_{FAt} = \frac{\tau_{FAt} \cdot b_2 \cdot b_3}{K_{tf}}$$

$\tau_{FAt} = 0,25 \cdot R_m = 112,5\ MPa$

$b_2 = 0,85$

$b_3 = 0,85$

$K_{tf} = 2,2$

$$\tau'_{FAt} = \frac{112,5 \cdot 0,85 \cdot 0,85}{2,2} = 36,95\ MPa$$

$$\sigma_{GP}^* = \sqrt{\sigma_a^2 + H^2 \tau_a^2} = \sqrt{(32,35)^2 + \left(\frac{67,7}{36,9}\right)^2 13,48^2} = 40,7\ MPa \leq \frac{\sigma_{a,lim}}{\eta}$$

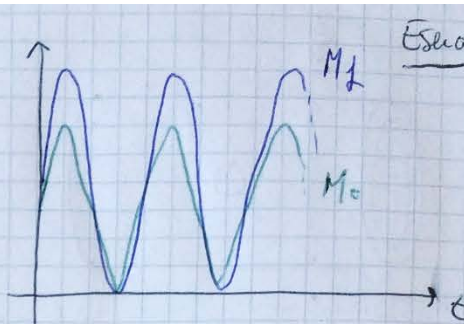
$$\eta = \frac{\sigma_{a,lim}}{\sigma_{GP}^*} = \frac{67,7\ MPa}{40,7\ MPa} = 1,66 < 2$$

Esercizio 6

Stesso esercizio di prima ma:

$$M_f = M_{f0}\sin(\omega t) + M_{f0}$$

$$M_t = M_{t0}\sin(\omega t) + M_{t0}$$

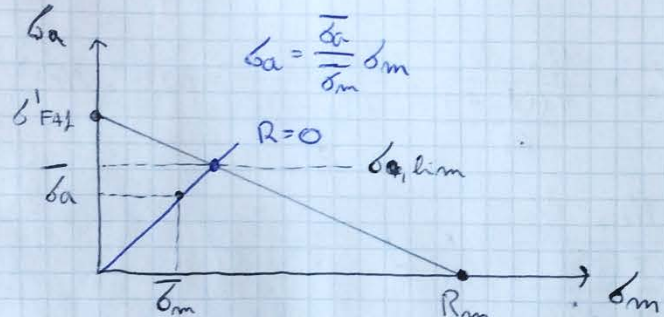


$\bar{\sigma}_a = 32{,}35$ MPa ; $\bar{\sigma}_m = 32{,}35$ MPa

$\bar{\tau}_a = 13{,}48$ MPa ; $\bar{\tau}_m = 13{,}48$ MPa

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_a^2} \qquad \text{dove} \qquad H = \frac{\sigma_{a,lim}}{\tau_{a,lim}}$$

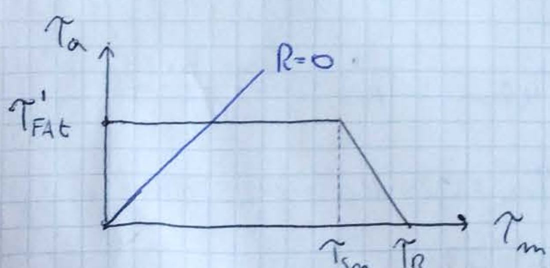
$\sigma'_{FAf} = 67{,}7$ MPa ; $\tau'_{FAt} = 36{,}95$ MPa



Goodman:

$$\begin{cases} \dfrac{\sigma_a}{\sigma'_{FAf}} + \dfrac{\sigma_m}{R_m} = 1 \\ \sigma_a = \sigma_m = \sigma_{a,lim} \end{cases}$$

$$\sigma_{a,lim}\left(\frac{1}{\sigma'_{FAf}} + \frac{1}{R_m}\right) = 1 \qquad \sigma_{a,lim} = 58{,}85 \text{ MPa}$$



$$\tau_{a,lim} = \tau'_{FAt} = 36{,}95 \text{ MPa}$$

$$\tau_{sm} = \frac{R_s}{\sqrt{3}} = 161{,}7 \text{ MPa}$$

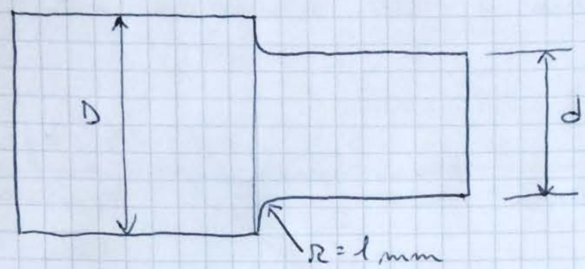
$$\tau_R = 0{,}77 \cdot R_m = 346{,}5 \text{ MPa}$$

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_a} = 38{,}83 \leq \frac{\sigma_{a,lim}}{\eta}$$

Condizione limite:

$$\eta = \frac{\sigma_{a,lim}}{\sigma^*_{GP}} = \frac{58{,}85 \text{ MPa}}{38{,}83 \text{ MPa}} = 1{,}52$$

Esercizio 7 - Albero con spallamento



$D = 30$ mm

$d = 25$ mm

$M_f = M_{fo} \sin(\omega t)$

$M_t = M_{to} = 150$ Nm

$M_{fo} = 100$ Nm

$R_t = 3{,}2$ μm

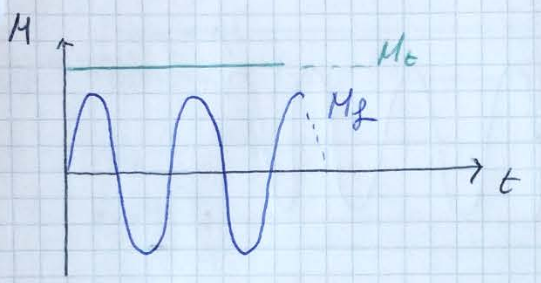
Materiale:

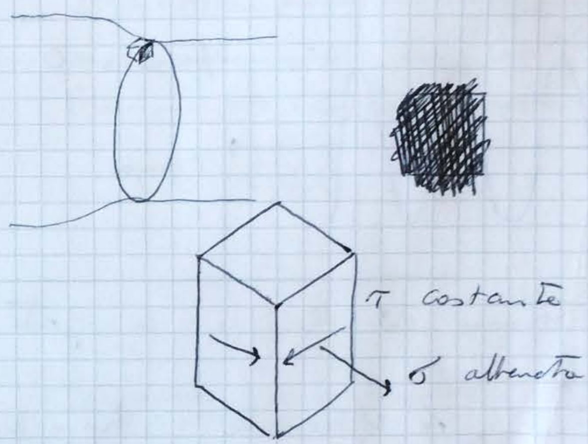
C40 temprato $R_m = 450$ MPa

$R_s = 280$ MPa

$M_f = M_{fo} \sin(\omega t)$ M_f alternato

M_t costante





$$\sigma = \frac{M_f \cdot d/2}{J} = \frac{M_f \cdot d/2}{\frac{\pi d^4}{64}} =$$

$$= \frac{M_{fo}\, d/2}{\frac{\pi d^4}{64}} \sin(\omega t) = 65{,}2 \sin(\omega t) \text{ MPa} \rightarrow \sigma_a = 65{,}2 \text{ MPa}$$

$$\tau = \tau_m = \frac{M_{to} \cdot d/2}{J_p} = \frac{M_{to}\, d/2}{\frac{\pi d^4}{32}} = 48{,}9 \text{ MPa}$$

Sforzi calcolati sul diametro nominale

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + H^2 \tau_m^2} \leq \frac{\sigma_{a,lim}}{\eta}$$

con $H = \frac{\sigma_{a,lim}}{\tau_{lim}}$; $\tau_{lim} = \frac{R_s}{\sqrt{3}} = 161{,}7$ MPa

$$\sigma_{a,lim} = \sigma'_{FAf} = \frac{\sigma_{FAf} \cdot b_2 \cdot b_3}{K_{ff}} =$$

$$= \frac{(0{,}5\, R_m)\, b_2 \cdot b_3}{K_{ff}} = 107{,}4 \text{ MPa}$$

$b_2 = 0{,}925$; $b_3 = 0{,}96$

$K_f = 2{,}2$ $(r/d = 0{,}04 ;\ D/d = 1{,}2)$

$K_{ff} = 1 + q(K_f - 1) = 1{,}86$

$q = \frac{1}{1+\sqrt{\frac{e}{r}}} = 0{,}7143$; $\sqrt{e} = 0{,}4$

$$\sigma^*_{GP} = \sqrt{\sigma_a^2 + \left(\frac{\sigma_{a,lim}}{\tau_{lim}}\right)^2 \tau_m^2} = 72{,}8 \text{ MPa}$$

$$\eta = \frac{\sigma_{a,lim}}{\sigma^*_{GP}} = \frac{107{,}4}{72{,}8} = 1{,}47 < 2$$