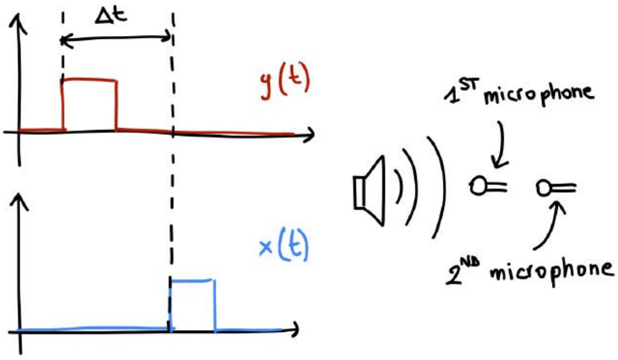
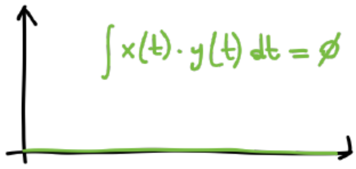
CROSS-CORRELATION AND AUTOCORRELATION

Cross-correlation

Let's imagine having two signals that have a trend like the one depicted in the following picture. We can imagine that they are signals coming from two different transducers: we have, for example, a sound speaker that emits a sound then we have two microphones. At first, of course, the sound reaches the first microphone and then it reaches the second one. We would like to know how long the sound takes to travel from the first microphone to the second one; in other words, we want to compute the time difference Δt between the two signals. To compute and know this delay of signal we can think of using a trigger but usually this is not done due to the fact that we could have a synthetic or a real sound and so this approach is not so effect; we want to find a more robust algorithm.

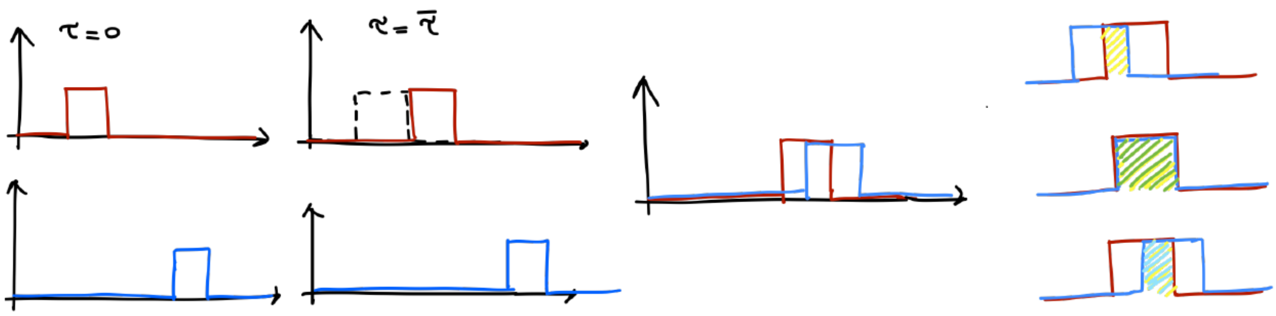


To compute the time delay between the two signals we can think of multiplying the signals and then doing the integral:



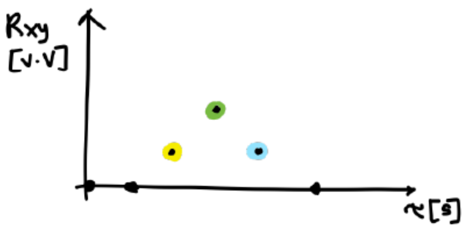
$$\int x(t) \cdot y(t) dt$$

The problem is that in the example just depicted the result of this operations will be zero since the multiplication between the two signals is zero. Thus, we need another strategy. We can think of shifting the first signal on the right side of a quantity τ which is a time dimensional value. Thanks to this strategy we end up with a value different from zero. Let's try to explain why through the example itself. Let's choose first a certain $\bar{\tau}$. Then for a $\tau = 0$ we have that $y(t)$ is in the same position as before and so the integral of the product between the two signals is zero. Then for $\tau = \bar{\tau}$ we have the signal $y(t)$ is moved forward of $\bar{\tau}$, since they two signals are not superimposed the result is still zero. Then for $\tau = 2\bar{\tau}$ we are still in the same situation and so the result is still zero. At a certain point the two signals will be superimposed and so the results will be a value different from zero.



To be more precise, by a mathematical point of view we are computing the following value that we call correlation:

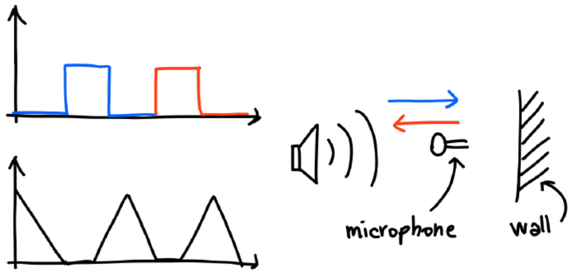
$$R_{xy} = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \cdot y(t+\tau) dt$$



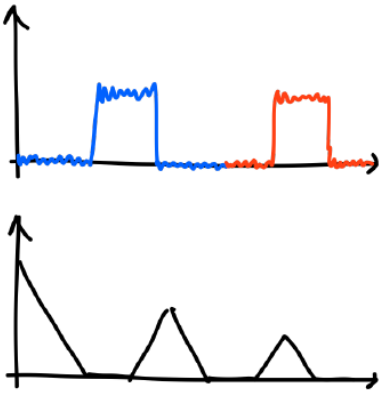
This concept gives us an idea on how the correlation between two signals is. By identifying the position in which we have the maximum value we can obtain the Δt. In the plot we can see that we don't have any correlation between the two signals until a certain point.

So, we have understood that the cross correlation can be used to determine the time delay between two signals.

Autocorrelation

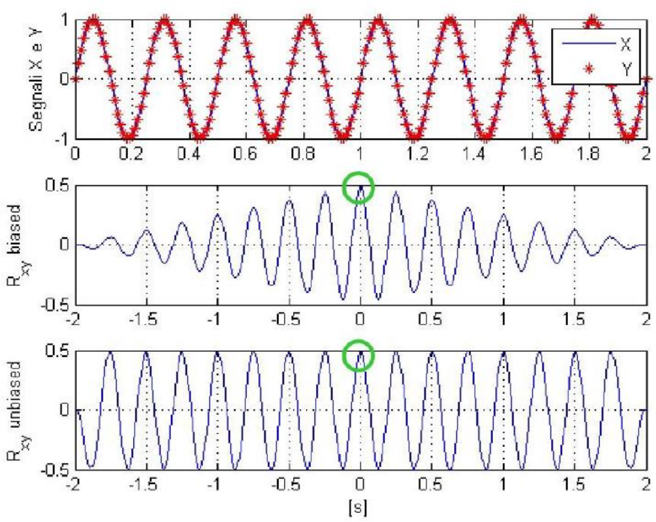


If now we imagine to eliminate the second microphone and add a wall after the first microphone we have a different situation: in this case we don't have two different signals but we have the same signals that hit the wall and comes back to be acquired a second to the microphone. In this case we could have a plot like the one shown in the picture. If we apply the concept described until now in this case, we end up with multiplying the signal by itself; this operation is called autocorrelation (it is a sort of artefact because I copy the original signal one and shift the first on). By a mathematical point of view the autocorrelation is defined as:

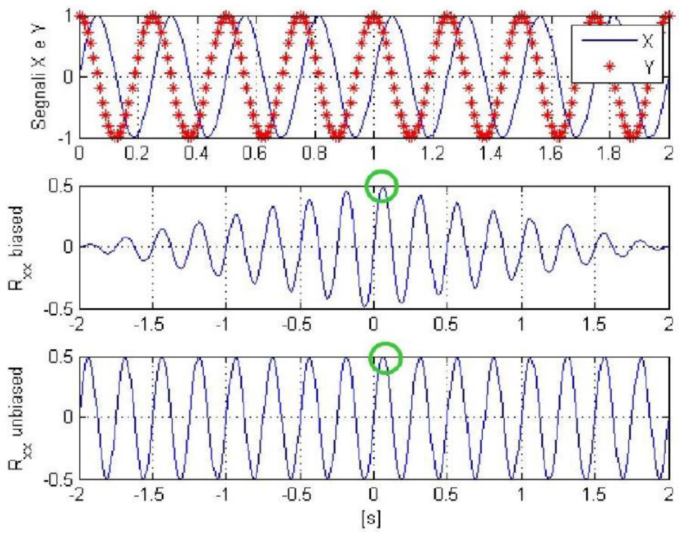


$$R_{xx} = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) \cdot x(t+\tau) dt$$

As opposed to the correlation in which we started with complete not superposition, in the autocorrelation we start with complete superimposition and then we decrease.

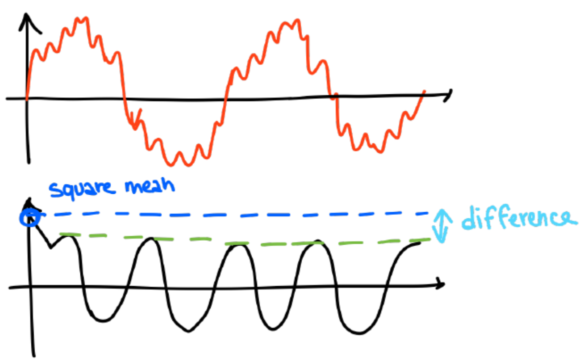


Graphical example of Auto-correlation



Graphical example of Cross-correlation

Autocorrelation: practical example

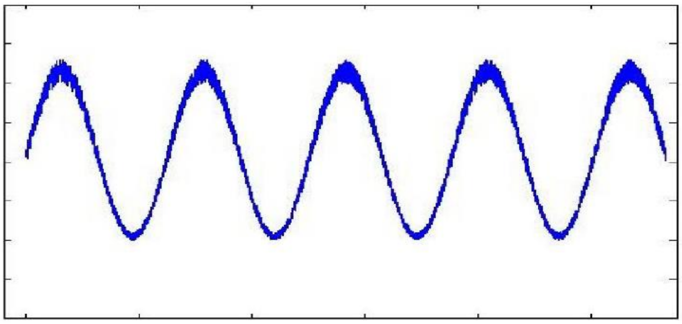


The autocorrelation can be used for several reason and aims:

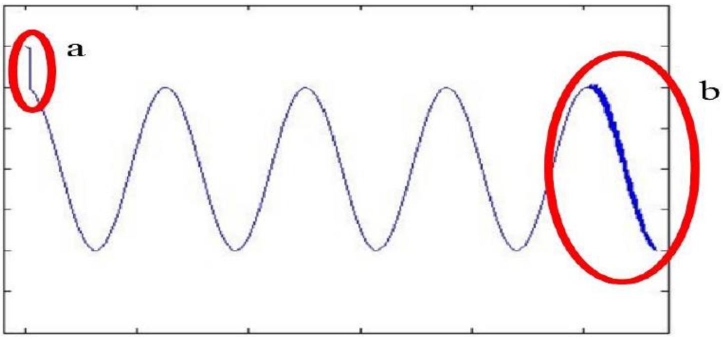
- Determine the periodicity of a particularly noise signal
- Determine how noisy is the signal
- Extract a harmonic signal free from noise if we have a stable signal affected by the noise. In order to do this, we must follow two steps:
  1. Apply an unbiased autocorrelation: after the unbiased autocorrelation we end with two main problems:

a. Problem A is generated by the complete overlapping of the two functions. In this case the noise of the two functions are completely overlapped and so the value of the integral is increases.
b. Problem B is due to the fact that the noise begins increasing because the length of the integration is not enough to reduce significantly its effect

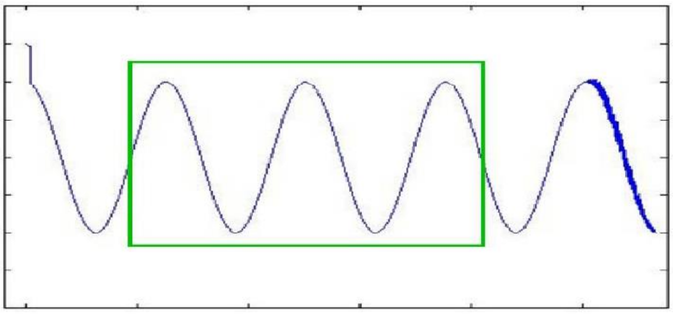
2. Extract a region not affected by the noise



Example of signal affected by noise



Unbiased auto-correlation on a signal affected by noise
and
Visualization of the two issues after unbiased auto-correlation



Extraction of the central part of the unbiased auto-correlation

Biased and unbiased correlation

In the previous definition provided of the correlation the factor of normalisation $1/T$ is indipendet from the variable τ: in this case we talk about the biased correlation. In this case if we are dealing with a finite signal, we have that as soon as τ increases the value of the biased correlation R_{xx} decreases. On the other hand, we can use the unbiased definition in which we take into account also τ in the normalisation factor ($\frac{1}{T-\tau}$) .

Transmission of data

Let's now try to focus our attention to one topic that we have introduced at the beginning of the course: what is a data bus? How the data are transmitted? The transmission changes with the respect to the type of measurements:

- Analogue: the data are transmitted as voltage proportional to the value, this transmission is able thanks to the wire
- Digital: in this type of transmission I use wires to transmit voltage but, in this case, we don't have a proportional voltage since the voltage has only two levels.

We must not believe that digital transmission is perfect. In fact, after we have 1 the voltage goes back to 0 but this is because we are sending a 0 or it is only to be considered as a separation to next transmitted 1? Thus, we need two types of information and so we need two couple of wires more that sends a service information: the start and the end of the acquisition. The second service information that is required is the Δt that divides every single information. This is still a little bit ambiguous because we now know how to indicate both the start and the stop but how can we know that the stop is a stop and not a start? We can solve this problem by adding a second *up* to indicates the stop so that we do not have any misunderstanding. We have now understood that the language is an important aspect and it's called protocol. With the same bus we can transmit different protocols.

So, we have defined the bus as a physical way in which we transmit a signal, the protocol is the meaning attributed to the bus. We need of course some agreement to have and use the same language.

We also need to try to avoid having too much wires and for this reason we can think of having a unique combination of data that works as out trigger so that we can eliminate two wires associated with the start of the signals. We can also think of doing the same thing for the end of the signal so that we end up with only having two wires. This transmission is called serial transmission. It was the first protocol used, the same used nowadays in the USB. By a general point of view this series are called hand shake between the transmitter and the receiver.

Difference between echo and ramble

We define echo as a reflection of sound that arrives at the listener with a delay after the direct sound. The delay is directly proportional to the distance of the reflecting surface from the source and the listener. Echo with respect to the ramble is able to distinguish the two sounds because they arrive to the humans' ears after $125\ ms$, whereas ramble doesn't distinguish the two sounds since the second sound arrives before $125ms$ and so the ears are not able to make this distinguish.

COMPITINO questions:

- I have a signal with an autocorrelation explain if the signs, is noisy or not noisy and why
- Given this function compute the correlation