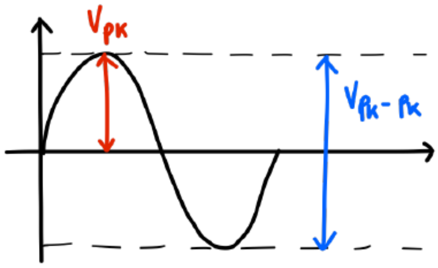
ROOT MEAN SQUARE

Introduction



Let's recall some basic nomenclature that will be extremely useful in this chapter:

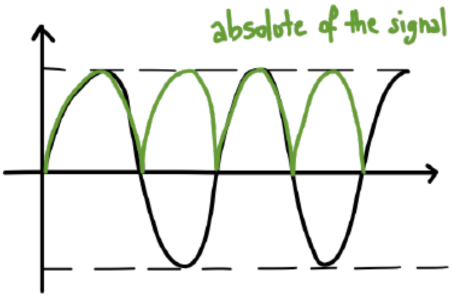
- Volt peak V_p= amplitude of the signal
- Volt peak to peak V_{pk-pk} = the difference between the minimum peak and the maximum peak.
- Mean value = it is the mean value of the signal, but, as we have previously stated it can be a tricky value, in fact if we think to the current we know that it has a trend of a sine wave as the one depicted in the picture. In this example the mean value (by a mathematical point of view) is of course 0 but if we put the finger into the socket, we won't' feel anything (even though the mean value is zero) since we'll feel the current. So, we conclude that this value is not a very reliable one to describe a dynamic signal.

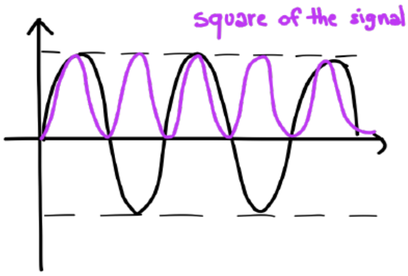
Though, if we are dealing with a signal that has a dynamic behaviour, we must use another concept that is related to the mean but that it's not the mean value. This concept is the RMS which stands for the Root Mean Square. We can consider the RMS for the dynamic behaviour as the equivalent of the mean value of a static signal. Let's try to understand how we reach the conclusion to use the RMS.

Approaches to handle a dynamic signal

In addition to this we must highlight that when we take into account a dynamic signal we are interested in both the negative and the positive part; so for us the positive and negative part have exactly the same weight and so we must find a way to quantify the phenomenon taking into account also this aspect. We can think of different ways to handle this aspect:

1. Absolute value = the easiest way that we can think of is taking the absolute value and then take the average this signal. In this case of course we end up with a value different from 0.
2. Square of the original wave = another approach that we can follow is taking the square of the original wave, this implicitly brings everything in the positive side.





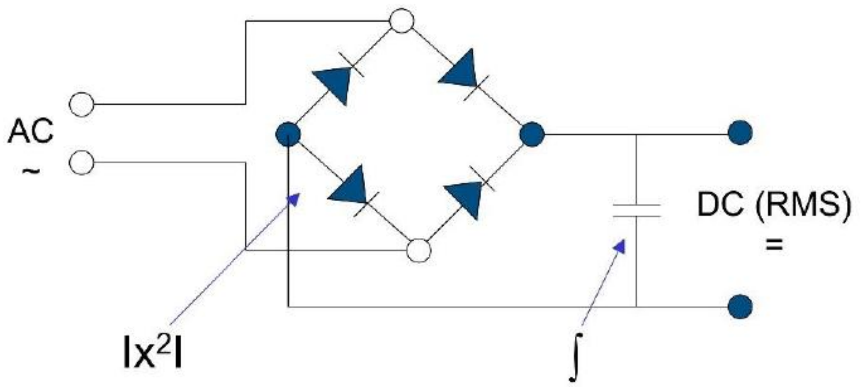
3. Electrical prospective = we can also consider an electrical prospective. In electrical fields, for alternate quantities, for instance AC voltage, the parameter for the measurement of the amplitude is the RMS. In fact, if we have a time varying signal at the inlet of a bridge diodes we'll have as the output a flat signal thanks to the rectification, since it "straightens" the direction of current. The bridge diodes is called rectifier and it's an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. The flat output represents the RMS voltage. We can use this definition that comes from the electrical fields also in out acoustic and mechanical fields since the transmission of the power along the hearing chain is very similar to the electrical transmission. Thus, as in electricity, also in acoustic the synthetic values of the AC quantities are expressed as RMS.

   The complete circuit for the RMS comprehends also a condenser (or capacitor) that cumulate the electrical charges. Since it is a analogical system it is not able to store in a memory the values computed

each time and when the circuit is unplugged the information is definitely lost. Thus, the condenser required a certain amount of time to reach the steady state condition. This amount of time is called time constant τ and it is defined as the time necessary to reach the 63% of the station value (which is the RMS value) after a step-excitation. The trend following during the charge will be always exponential. If we increase the capacity of the condenser, it will be able to store more data but its promptness decreases, so we need to define some standard so that we can compare the results.
Now day we use digital device that have no problems in term of promptness but we still need to compare our results with the analogic standards present in literature, so to recreate exactly the behaviours that we would have in the analogical device with a digital one we change the length of the window. In the standards three types of window have been defined:

- Fast = 0,125 s: it was chosen since it's the limit thanks to which we identify the difference between ramble and eco sound.
- Low = 1,00 s
- Impulse

All of these three time-constants are from a certain point a low pass filter. In general, if the time constant is low the RMS goes up and goes down faster, whereas if it's high the RMS goes up and down low. An important aspect to cover is do I need such a huge amount of data? No because, as long as my window is a low pass filter, since it eliminates all the fluctuation, all the last part of the bandwidth is useless so instead of keeping all the signal we can think of taking into account only the first part. So, we can reduce the number of data without losing anything.



RMS

Let's now focus our attention onto the last approach that we have just defined according to which we consider the RMS as the indicator of the amplitude for dynamic quantities. The English definition of the RMS is focused on the mathematics of the problem: root, mean and square. On the other hand, the Italian or Latin definition of this parameter are a little bit more meaningful since it is called *valore efficace* that in English would sound approximately like *effective* value that is a value proportional to the effect induced by the AC phenomenon. The formula of the RMS can be defined in two different ways, following two difference points of view:

- Mathematical point of view:

$$RMS = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$

- Measurement point of view:

$$RMS = \sqrt{\frac{\sum x_i^2}{N}}$$

Since we are computing $x^2(t)$ we can say that:

- The signal becomes always positive
- The mean value of the signal is offset in the positive semi-plane

Let's now notice some key points of the RMS:

1. The formula just provided is valid only for the sinewave
2. The RMS is an average quantity, not an instantaneous value, and for this reason it doesn't make any sense to calculate the RMS of one sample or even on a period much shorter than the time of the cycle of an AC wave.
3. The RMS of a DC value is the DC value itself
4. The RMS is always non-negative, so it will always be positive or at least equal to zero
5. The RMS of a sine is equal to the peak value divided by $1/\sqrt{2}$

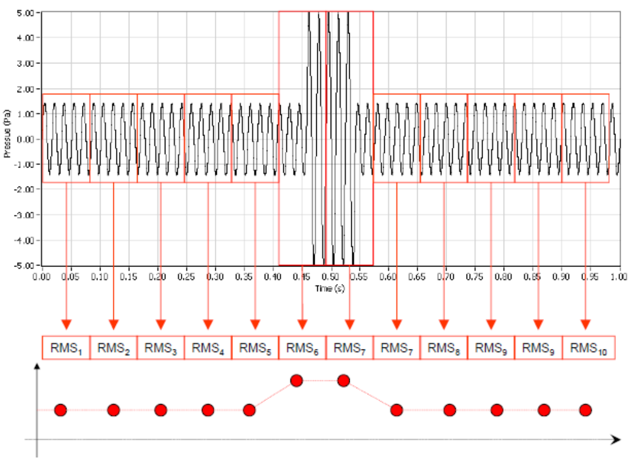
It's important to highlight that the RMS of a time changing signal can be not univocal due to 2 main reasons:

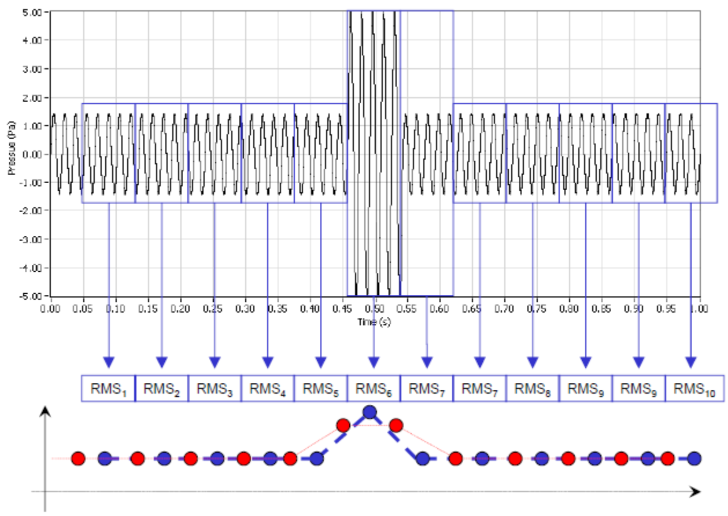
a) Different processing starting point
b) Different length of the window

In order to understand better these two aspects, let's take into account the example of what happen in a burst, which is a typical varying signal that have an almost instantaneous increase of amplitude that is maintained for a certain amount of time, and then comes back to lower

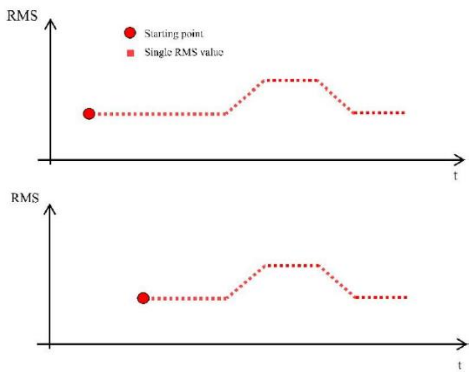
Different processing starting point values

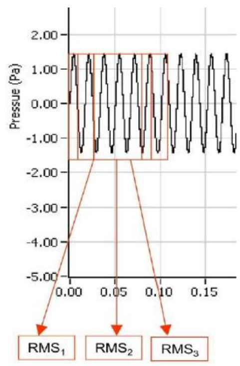
Let's assume that the calculation of the RMS is made by processing one discrete window per time. Depending on what region of the signal fells into the window we can compute different values of RMS, of course the region of the signal fells into the window depends on the starting point. In the following two imagine I have a clear example of an RMS of a burst computed with one discrete window per time with different starting points.





The solution to solve this problem is choosing a moving window which is a window that is moving, and which start at each sampling points of the signal. Thanks to this technique after a certain amount of time depending of which sample, we start, all the data will be the same and so reliable one to the other. Of course, this process can be done just in the digital since we are dealing with a discrete domain and not in analogical since we have a continuous domain. The main drawback of this process is the amount of information and the computing process.





Different length of the window

Regarding this second problem we must say that is has not been technically solved, but simply it was agreed that some standardized interval had to be chosen. To be more precise a time constant has to be chosen. The standard has established 3 time constants, the one already introduced.

<u>Exercise</u> (compitino): we have a burst (a sine with a short duration) with a duration of $1s$ as the one shown in the picture. Its amplitude is given and it's $V_{pk} = 20m/s^2$. Knowing that the amplitude of the noise is $V_{noise} = 1\,m/s^2$, $s$ketch qualitatively the RMS time history for three different time constants: $125ms$, $250ms$and $100ms$.

In order to proceed with the sketch, we need first to compute the RMS value of both the noise and the burst. To do that we must multiply each peak per $1/\sqrt{2}$:

$$RMS_{noise} = \frac{V_{pk}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0{,}707 \qquad RMS_{burst} = \frac{V_{pk}}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14{,}14$$

Now, regarding the burst we must compute the value reached when the constants is reached:

$$63\% \cdot RMS_{burst} = \frac{63}{100} \cdot 14{,}14 = 8{,}91$$

Now we must draw the RMS time history in the three different cases:

- Since we know that the first time constant is $0{,}125s$ and we know that the duration of the burst is 1s we can conclude that we'll need 1 period to reach the steady state condition
- if we have a time constant of $250ms$ we'll need 2 period to reach the steady state condition
- if we have a time constant of 1000ms we'll never reach the steady state condition

So, in conclusion, we can say that the number of periods required to reach the steady state condition can be computed as the ratio between the total length of the burst and the time constant. The final plot will be: