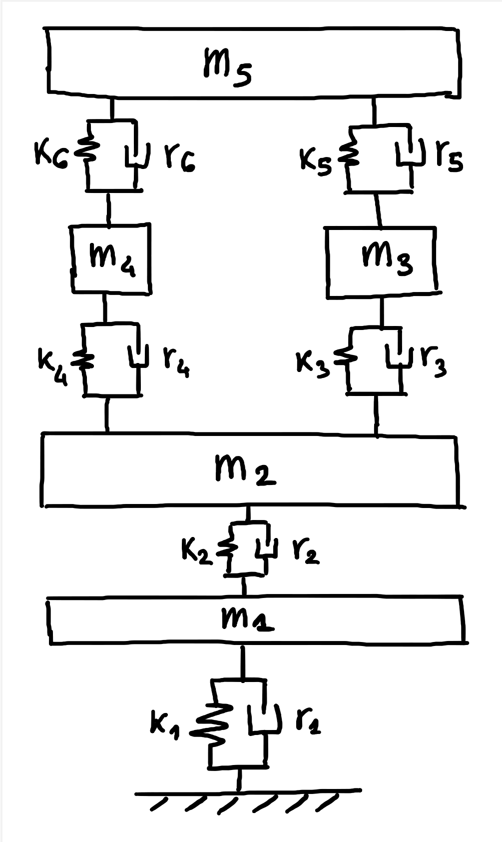
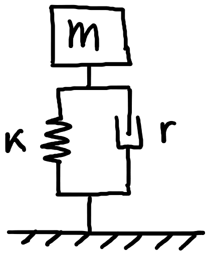
FRF AND EXCITATIONG TECHNIQUES

Introduction

Let's imagine that we have a quite complex mechanical system like the one depicted in the picture on the right. Usually when we are dealing with this kind of mechanical system, we always want to discover some feature of the system itself and generally we want to compute its natural frequency.



We know that the natural frequency of a simple system (1 degree of freedom), like the one in the picture below, made only of a mass, a damper and a spring can be computed as the square root of the ratio between the spring and the mass:



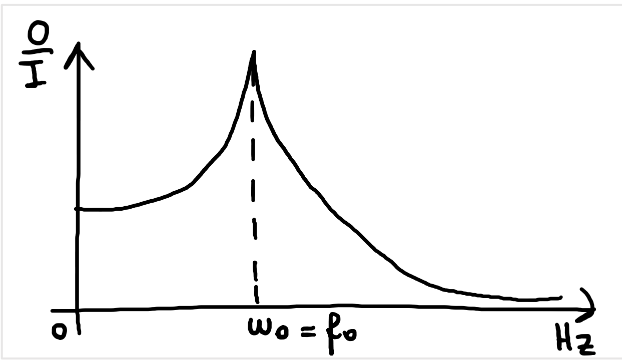
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Since we are in the measurement course we want to understand if we can measure the natural frequency or at least take the measurements of the different parameters that are presented in the equation. So, the question that we now would like to answer is how we can compute the parameters required to compute the natural frequency (mass and spring). We know for sure that we can measure the mass thanks to a balance, for example. On the other hand, we are not so sure to be able to measure the stiffness of the spring since it will depend also on other aspects like how it is mounted. Maybe we can define it thanks to some experiments, but the value will never be precise for the reasons already explained. If then we would like to measure also the damper to know all the parameters of the system (even if it's not necessary to compute the natural frequency), we can say that it is almost impossible measuring it.

Let's make an example to clarify everything. Let's consider a personal computer, and let's image to hit the screen. We are now interested in measuring the parameters related to the screen that is vibrating. Of course, we can measure the mass of the screen by unscrewing it from the keyboard and using a balance. Maybe we can also compute the stiffness of the spring by studying for example the screw, but for sure we have no idea on how we can measure the damping.

So, we have understood that to determine the natural frequency, measuring the parameters of system is not effective. We can think of using a different approach and have a look at a plot called frequency response function.

Frequency response function



So, we can think of studying the natural frequencies by using another approach. We, in fact, can think of using a graph, a spectrum, which is the responses of the system to an input; this response is called frequency response function (FRF). The idea is that I excite the mechanical system at different input frequency, and I obtain as the output a response that changes with the frequency.

Once we have obtained the spectrum, we can identify the natural frequency of the system by identifying the frequency at which we have the peak.
The plot that we have obtained is a spectrum, as already mentioned. To be more précised it is the spectrum obtained by computing the ration between the spectrum of the output and the spectrum of the input.
Until now in the spectrum we have had in the vertical axis a specific quantity, an amplitude expressed in volt, for example, whereas now in the vertical axis we have a ratio and not a pure value. The ratio, as already mentioned, is obtained by dividing the output over the input. For this reason, once again, this plot can be considered as the combination of two spectra or at least the combination of two different behaviours.
Usually to obtain the output used to compute the FRF some experiments are carried out. These experiments consist in the excitement with a controlled input. These controlled inputs could be of different types; the main ones are:

1. Step sine (or harmonic excitation)
2. Swept sine
3. Impulsive
4. White noise

Step sine technique

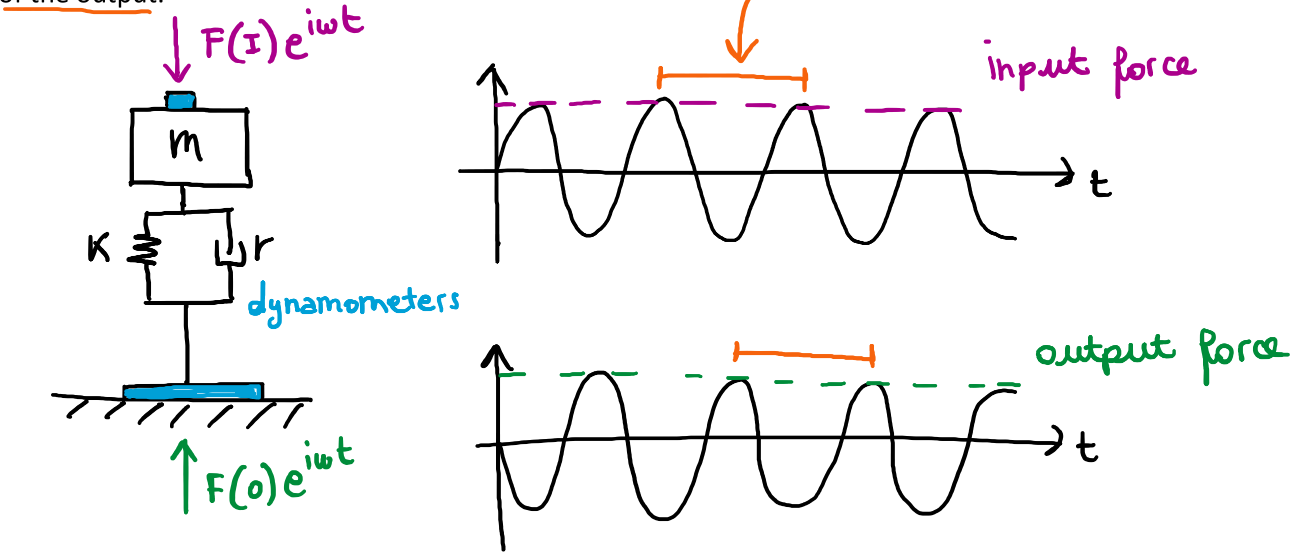
In this first case a simple sine wave is used to excite the system. Usually it is provided to the system thanks to the use of a vibrating machine on which the system is mounted. In order to study the response of different frequency, the frequency of the input must be changed following a discrete step previously defined. It's important to choose correctly the step of the frequency so that none frequency of interest is missed.
So, let's go back to our simple mechanical system with the spring and the damper; and let's understand how we can proceed in this case with the step sine approach. First, we must highlight that we could have two different case:

a) In this first case I know the input force
b) In this second case we don't know the input force. In this case a dynamometer placed on the mass is required so that we can measure and know also the input force

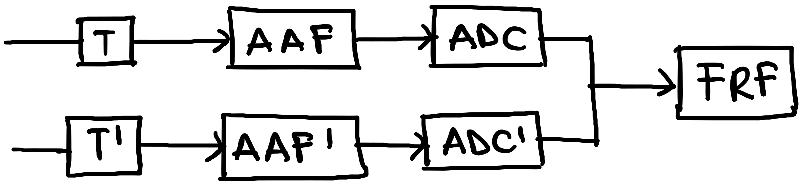
Anyway, in both cases thanks to a dynamometer that is usually placed on the ground we can acquire the output force. Now we have all the ingredients to compute the ratio between output and input.
As already stated, we need a force that changes its frequency and for this reason we usually have an exponential force and not a static one. So, we have a force that has a dynamic behaviour so that we can analyse and study all the interested frequency. So, to repeat once again we take measurement for different frequency; for each frequency we have an input and an output and then we can compute the ratio.
To sum up this approach we can say that we need to follow these steps:

- Choose a frequency variable input force
- Measure the output associated to that frequency
- For each frequency we must compute the ratio
- We finally can draw the plot

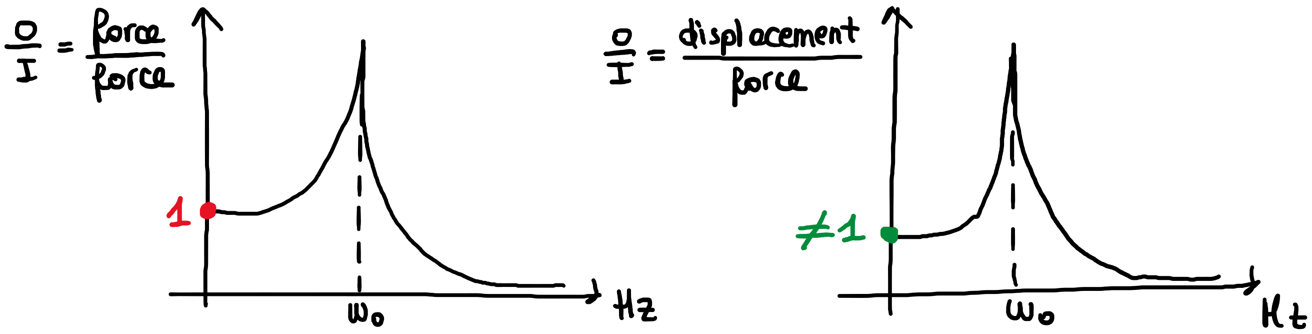
We can now draw once the scheme of our mechanical system with the plot of both the input and the output force. It's important to highlight that if the system is linear the period of the input signal is equal to the period of the output.



So now, with the introduction and the use of the FRF, we need two channels to carry out the measurement: one for the input and one for output; for this reason, we'll have two chains of measurement that are then connected together to have the FRF.



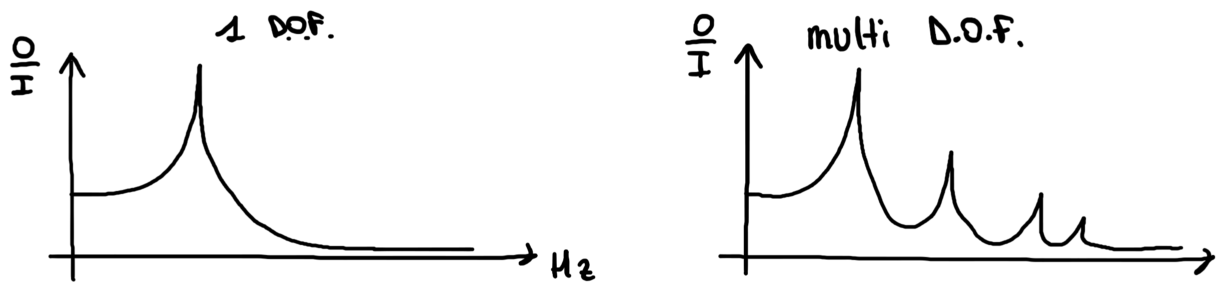
Until now we have discussed about the case in which the output is the force (the same "type" of the input), of course if we are interested in the displacement as the output and not in the force, we can think of placing a displacement transducer instead of the dynamometer. So, let's point out that the $F_o/F_i = 1$ for $\omega = 0$ as long as we use force over force, so in case of having displacement over force this value will be different from 1. We can now draw once again the scheme of our mechanical system with the plot of both the input and the output force. It's important to highlight that if the system is linear the period of the input signal is equal to the period of the output.



Under a theoretical point of view, I could use any amplitude of the force but by a practical and measurement point of view we must be sure that the input and the output are higher than the sensitivity so that they can be acquired correctly. Usually we could have two critical situations:

1. If we are near the $\omega = 0$ we could have forces too small to be acquired correctly (they could be affected by noise)
2. If we are near the peak, and so near the natural frequency ω_0, we could have force too high that could damage our system

In general, if our system has only one degree of freedom what we have done until now gives us a comprehensive and good result but when I have a multi degree of freedom we are not sure that following this path we'll have a reliable result. This is due to the fact that in this technique I need a frequency step that can actually affect our FRF: I could for example choose a frequency step that is bigger than the different between the first and the second natural frequency and so I could obtain a distorted value of the second natural frequency or I could complete lose the information of the second natural frequency. For this reason, we are going to introduce the other approaches that can help me calculate the FRF with a trade-off between accuracy, cost and speed (usually asked in the second midterm exam).



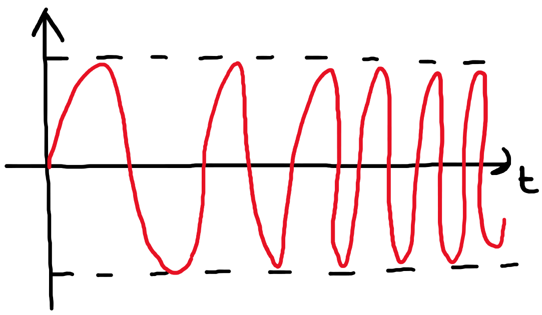
So, we have understood that his technique has a very big limitation, but on the other have it has a quite important advantage: in this approach we measure in steady state condition.
The technique that we have described until now and that it's usually used in a single degree of freedom system.

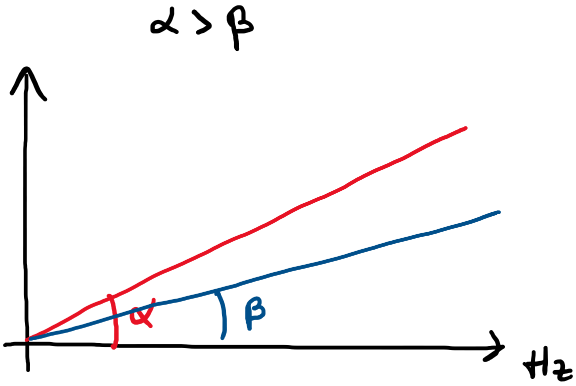
Swept sine technique

In order to overcome the limitation of the step sine approach we can think of use another method that is called swept sine. This technique is based on the idea of exciting the system with a harmonic function which frequency changes continuously in time as shown in the picture on the right in the time domain. In other words, we are exciting the system with a sine wave whose period keeps on decreasing. Thanks to this we remove the problem of the resolution frequency since we excite all the frequency of the range. The main drawback of this method is that we are never in the steady state condition and so we actually never reach the pure FRF since it is defined as steady state. In order to improve this aspect, we can think of setting a very low rate of increase.

The swept sine function is a function of the slope frequency and so in order to be very accurate we need to take a long frequency range which means having a decrease of the slope.

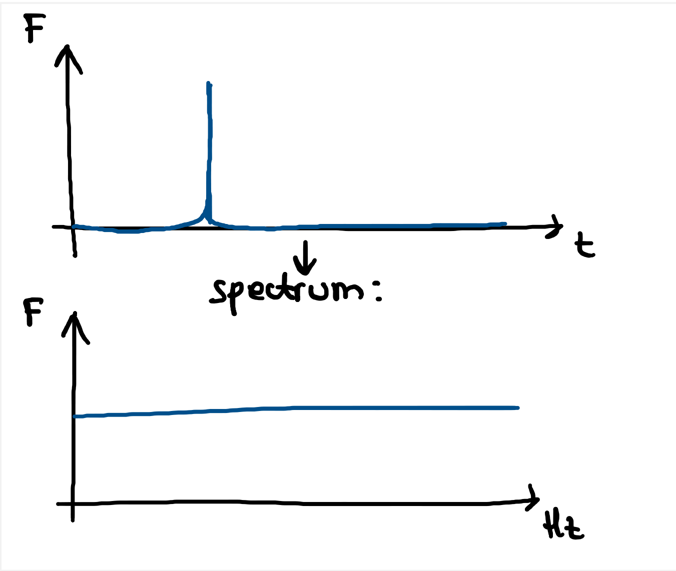
In order to have the best of both worlds usually a speed overview is done with a swept sine of intermediate level and then it's possible to go deeper in the details and go around the peaks under steady state with the step sine approach.





Impulsive technique

Until now we have talked about natural frequency, but we haven't defined it yet. A natural frequency is the frequency at which it is easy for the system to vibrate. We can use this definition and observation to develop a new technique to obtain the FRF. We can think of excite the system with an impulse since it leads us to cover all the frequencies in a very short time.



Its advantages are that it is the fastest, we don't have to make any choice about the resolution and so the resolution will be for sure $1/T$. The problem is that is not steady state at all, and it is only transient. It is a very effective technique, but it cannot lead us to have a steady state response function, it can be used to identify where the natural frequencies are.

The duration of the acquisition is an important parameter to highlight since it depends on the resolution itself. So, the key point is that I cannot acquire whenever I want but I have to respect a condition according to which the system has already finished to vibrate before I do another acquisition. So, if the frequency resolution is $0{,}01 Hz$, I have to wait at least $100 s$ before do another acquisition.

White noise technique

There is another technique that do not generate a harmonic function, but it generates some noise. A random signal, white noise, which covers a lot of frequencies. Usually this technique requires following averaging procedure.

Adding notes

We could have two different problems:

a) Problems related to two channels = multiplex ADC that shares one converter for both channels can lead to small delay
b) Transducer = the dynamic behaviour of the transducer can be a problem when we take a ratio

	Advantages	Disadvantages
Step sine	• We are in the steady state condition	• Problem of the resolution (discrete frequency step) • Very time consuming
Swept sine	• no problem on frequency resolution since I excite all the frequencies	• we are never in the steady state condition • FRF almost never reached because we keep on increasing the frequency (this can be reduced by setting a very low rate of increase)
Impulse	• it's fastest • It is cheap	• It is transient and not steady state at all and so FRF is never reached • Maybe difficult to have an adequate energy input without damaging the system
White noise	• Easy to implement	• It is transient and not steady state at all and so FRF is never reached • Could not cover the frequency of interest because it’s a random signal