Integrali definiti

1. Calcolare i seguenti integrali definiti:

1) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \log(1+\sin x + |\sin x|)\, dx$

2) $\displaystyle\int_{0}^{\frac{\pi}{4}} (1+\tan^2 x) \arctan(1+\tan x)\, dx$

3) $\displaystyle\int_{1}^{4} \frac{\sqrt{x-1}}{x+8}\, dx$

4) $\displaystyle\int_{0}^{\frac{\pi}{6}} \sin 6x e^{\sin 3x}\, dx$

5) $\displaystyle\int_{1}^{e} \frac{\log x}{x\sqrt{1+\log^2 x}}\, dx$

6) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x \log(\sin x)}{\sin x}\, dx$

7) $\displaystyle\int_{1}^{e} (1+\log x)\cos(x \log x)\, dx$

8) $\displaystyle\int_{0}^{1} x^5 e^{x^2}\, dx$

9) $\displaystyle\int_{0}^{\frac{\sqrt{3}}{2}} \sqrt{3-x^2}\, dx$

10) $\displaystyle\int_{-1}^{0} \frac{\sqrt{1+x}}{2+x}\, dx$

11) $\displaystyle\int_{0}^{\log 6} \frac{1}{\sqrt{e^x+3}}\, dx$

12) $\displaystyle\int_{0}^{8} \arctan \sqrt[4]{1+x}\, dx$

13) $\displaystyle\int_{0}^{(\frac{\pi}{3})^2} \frac{1}{\cos^2 \sqrt{x}}\, dx$

14) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{4-|\sin x|\sin x}\, dx$

15) $\displaystyle\int_{0}^{4} \big|\arctan(1-x)\big|\, dx$

2. Calcolare l'area dell'ellisse di equazione $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

Soluzioni.

1. 1) $\dfrac{1}{2}(3\log 3 - 2)$, si pone $\sin x = t$.

2) $2\arctan 2 - \dfrac{\pi}{4} - \dfrac{1}{2}(\log 5 - \log 2)$, si pone $1 + \tan x = t$.

3) $2\sqrt{3} - \pi$, si pone $\sqrt{x-1} = t$.

4) $\dfrac{2}{3}$, si pone $\sin 3x = t$.

5) $\sqrt{2} - 1$, si pone $\log x = t$.

6) $-\dfrac{1}{8}\log^2 2$, si pone $\log(\sin x) = t$.

7) $\sin e$, si pone $x\log x = t$.

8) $\dfrac{1}{2}e - 1$, si pone $x^2 = t$.

9) $\dfrac{3}{8}(\sqrt{3} + \pi)$, si pone $\dfrac{x}{\sqrt{3}} = t$.

10) $2 - \dfrac{\pi}{2}$, si pone $\sqrt{1+x} = t$.

11) $\dfrac{1}{\sqrt{3}}\log(2+\sqrt{3})$, si pone $\sqrt{e^x + 3} = t$.

12) Ponendo $\sqrt[4]{1+x} = t$, l'integrale dato diventa: $\displaystyle\int_1^{\sqrt{3}} 4t^3 \arctan t\, dt$, integrando per parti si trova: $\displaystyle t^4 \arctan t\Big|_1^{\sqrt{3}} - \int_1^{\sqrt{3}} \frac{t^4}{1+t^2}\, dt = 3\pi - \frac{\pi}{4} - \int_1^{\sqrt{3}} \left(t^2 - 1 + \frac{1}{1+t^2}\right) dt = \frac{2}{3}(4\pi - 1)$

13) Ponendo $\sqrt{x} = t$, l'integrale dato diventa: $\displaystyle\int_1^{\frac{\pi}{3}} \frac{2t}{\cos^2 t}\, dt$, integrando per parti si trova: $\dfrac{2\sqrt{3}}{3}\pi - 2\log 2$

14) Ponendo $\sin x = t$, l'integrale dato diventa: $\displaystyle\int_{-1}^{1} \frac{1}{4 - t|t|}\, dt = \int_{-1}^{0} \frac{1}{4+t^2}\, dt + \int_0^1 \frac{1}{4-t^2}\, dt = \frac{1}{8}\pi + \frac{1}{4}\log 3$

15) L'integrale dato vale: $\int_0^1 \arctan(1-x)\,dx - \int_1^4 \arctan(1-x)\,dx$, integrando per parti si trova: $F(x) = \int \arctan(1-x)\,dx = -(1-x)\arctan(1-x) + \frac{1}{2}\log(1+(1-x)^2)$, allora l'integrale dato vale $F(1) - F(0) - F(4) + F(1) = \frac{\pi}{4} - \frac{1}{2}\log 2 + 3\arctan 3 - \frac{1}{2}\log 10$

2. L'area vale $4\int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx = 6\pi$, si pone $x = 2\sin t$.