# Equazioni di Laplace

Laplace

$$\begin{cases} -\Delta u_i = 0 & in\ \Omega \\ u_i = g_i & in\ \Gamma_i \\ u_i = 0 & in\ \dfrac{\partial\Omega}{\partial\Gamma_i} \end{cases}$$

$$u_1(x,y) = \sum_{n=1}^{+\infty} A_n^1 \sin\left(\frac{n\pi}{L}x\right) \sinh\left[\frac{n\pi}{L}(H-y)\right] \frac{1}{\sinh\left(\frac{n\pi H}{L}\right)}$$

$$A_n^1 = \frac{2}{L}\int_0^L g_1(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

$$u_2(x,y) = \sum_{n=1}^{+\infty} A_n^2 \sin\left(\frac{n\pi}{L}x\right) \sinh\left[\frac{n\pi}{L}y\right] \frac{1}{\sinh\left(\frac{n\pi H}{L}\right)}$$

$$A_n^2 = \frac{2}{L}\int_0^L g_2(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

$$u_3(x,y) = \sum_{n=1}^{+\infty} A_n^3 \sin\left(\frac{n\pi}{H}y\right) \sinh\left[\frac{n\pi}{H}(L-x)\right] \frac{1}{\sinh\left(\frac{n\pi L}{H}\right)}$$

$$A_n^3 = \frac{2}{H}\int_0^H g_3(y) \sin\left(\frac{n\pi}{H}y\right) dy$$

$$u_4(x,y) = \sum_{n=1}^{+\infty} A_n^4 \sin\left(\frac{n\pi}{H}y\right) \sinh\left[\frac{n\pi}{H}x\right] \frac{1}{\sinh\left(\frac{n\pi L}{H}\right)}$$

$$A_n^4 = \frac{2}{H}\int_0^H g_4(y) \sin\left(\frac{n\pi}{H}y\right) dy$$

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|---|---|
| Condizione di Dirichlet | $u = g$ |
| Condizioni di Neumann | $-\vec{\nabla} u \cdot \hat{n} = h$ |
| Condizioni di Robin | $-\vec{\nabla} u \cdot \hat{n} - \alpha u = h$ |

<u>Separazione variabili in coordinate polari (cerchio)</u>:

$$\begin{cases} x = R\cos\vartheta \\ y = R\sin\vartheta \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

$$\vartheta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\Delta u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \vartheta^2}$$

$$u(r,\vartheta) = A_0 + \left[\sum_{n=1}^{\infty} A_n \cos(n\vartheta) + \sum_{n=1}^{\infty} B_n \sin(n\vartheta)\right]\left(\frac{r}{R}\right)^2$$

$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\vartheta)d\vartheta$$

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\vartheta)\cos(n\vartheta)\,d\vartheta$$

$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\vartheta)\sin(n\vartheta)\,d\vartheta$$