

Integrali di funzioni razionali

1. Calcolare i seguenti integrali di funzioni razionali:

$$1) \int \frac{1}{x^2 - 3x + 2} dx$$

$$2) \int \frac{3x - 4}{x^2 - 5x + 6} dx$$

$$3) \int \frac{x + 3}{x^2 - 2x + 1} dx$$

$$4) \int \frac{x + 1}{(3x + 2)^2} dx$$

$$5) \int \frac{x + 2}{2x^2 + 4x + 3} dx$$

$$6) \int \frac{x^3 + x^2 - x - 2}{x^2 + x + 1} dx$$

$$7) \int \frac{1}{x^3 - 1} dx$$

$$8) \int \frac{1}{(x + 1)(x^2 - 1)} dx$$

$$9) \int \frac{x^6 - 1}{x - 1} dx$$

$$10) \int \frac{1 + 2x^2}{x^4 - 1} dx$$

2. Calcolare i seguenti integrali:

$$1) \int \frac{1}{e^x + 1} dx$$

$$2) \int \frac{1 + 2e^x}{e^{2x} - 1} dx$$

$$3) \int \frac{\sqrt{x}}{2 + \sqrt{x}} dx$$

$$4) \int \frac{1}{x\sqrt{x + 4}} dx$$

$$5) \int \tan^3 x dx$$

3. Calcolare i seguenti integrali di funzioni trigonometriche:

1) $\int \frac{1}{2 + \cos x} dx$

2) $\int \frac{1}{\sin x} dx$

3) $\int \frac{1}{1 + \sin x} dx$

4) $\int \frac{1}{\cos x} dx$

5) $\int \frac{1}{\sin x + \cos x} dx$

Soluzioni.

1. 1) $-\log|x-1| + \log|x-2| + c$

2) $-2\log|x-2| + 5\log|x-3| + c$

3) $\log|x-1| - \frac{4}{x-1} + c.$

4) $\frac{1}{9}(\log|3x+2| - \frac{1}{3x+2}) + c.$

5) $\frac{1}{4}\log(2x^2 + 4x + 3) + \frac{1}{\sqrt{2}}\arctan\sqrt{2}(x+1) + c$

6) $\frac{1}{2}x^2 - \log(x^2 + x + 1) - \frac{2}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + c$

7) $\frac{1}{3}\log|x-1| - \frac{1}{6}\log(x^2 + x + 1) - \frac{1}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}} + c$

8) $-\frac{1}{4}\log|x+1| + \frac{1}{2}\frac{1}{x+1} + \frac{1}{4}\log|x-1| + c$

9) $\frac{1}{6}x^6 + \frac{1}{5}x^5 + \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$

10) $\frac{3}{4}(\log|x-1| - \log|x+1|) + \frac{1}{2}\arctan x + c$

2. 1) $x - \log(e^x + 1) + c$

2) $-x + \frac{3}{2}\log|e^x - 1| - \frac{1}{2}\log(e^x + 1) + c$

- 3) $2x - 4\sqrt{x} + 8\log(\sqrt{x} + 2) + c$, si pone $\sqrt{x} = t$.
- 4) $\frac{1}{2}(\log|\sqrt{x+4} - 2| - \log(\sqrt{x+4} + 2)) + c$, si pone $\sqrt{x+4} = t$.
- 5) $\frac{1}{2}(\tan^2 x - \log(1 + \tan^2 x)) + c$, si pone $\tan x = t$.

3. 1) $\frac{2}{\sqrt{3}} \arctan\left(\frac{1}{\sqrt{3}} \arctan \frac{x}{2}\right) + c$

2) $\log\left|\tan \frac{x}{2}\right| + c$

3) $-2\frac{1}{1 + \tan \frac{x}{2}} + c$

4) $\log\left|\frac{1 + \tan \frac{x}{2}}{1 - \tan \frac{x}{2}}\right| + c$

5) $\frac{1}{\sqrt{2}} \log\left|\frac{\tan \frac{x}{2} - 1 + \sqrt{2}}{\tan \frac{x}{2} - 1 - \sqrt{2}}\right| + c$